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VERTICAL SHIP MOTIONS IN SHALLOW WATER

BY

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DECLARATION OF CANDIDATE

I, Kevan Richard Kenilworth Blake, hereby declare that this thesis is my own work and that it has not been submitted for a degree at another university.

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ABSTRACT

With the increase in ship sizes, there has been an increased interest in the prediction of ship motions in shallow water, where the possibility of grounding becomes a problem. Theoretical equations governing the ship's motions have been formulated involving various hydrodynamic coefficients. In this thesis these coefficients have been found experimentally for a range of water depths and wave periods. The methods for solving the equations of motion theoretically are introduced and discussed. The equations of motion are solved using the coefficients, found experimentally, to give results in the form of motion response amplitude operators. These are compared against other authors' results derived either theoretically or experimentally.

It was found that the hydrodynamic coefficients increase rapidly with decrease in water depth. Relatively good agreement between the empirical results of this thesis and results of other authors would indicate that the theoretical formulae do represent the coefficients to which they are claimed to approximate, for at least the case of zero or low forward speeds.

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LIST OF SYMBOLS AND ABBREVIATIONS

a	= axial distance from centre of gravity to spring supports (m)
a	= radius of a cylinder (m)
A_{33}	= virtual mass in forced heaving
A_{55}	= virtual moment of inertia in forced pitch
A_{35}, A_{53}	= coupling coefficients linking the two modes of motion
AMP.	= amplitude
b	= damping factor
B	= extreme breadth (m)
B_{33}	= heave damping coefficient
B_{55}	= pitch damping coefficient
B_{35}, B_{53}	= coupling damping coefficients linking the two modes of motion
c	= damping of an oscillating motion
c	= wave celerity ($m.s^{-1}$)
C_b	= block coefficient
C_c	= critical damping of an oscillating motion
C_W	= water line plane area coefficient
C_{33}	= heave hydrostatic restoring force

C_{55} = pitch hydrostatic restoring force

C_{35}, C_{53} = coupling restoring force coefficients linking the two modes of motion

d = water depth (m)

dwt = dead weight tonnes (t)

e_h = phase difference between wave and exciting force (rad)

e_p = phase difference between wave and exciting moment (rad)

F = amplitude of exciting force (N or N.m)

Fr = Froude number

F_{aft} = force on aft cantilever (N)

F_{fore} = force on fore cantilever (N)

F_3 = heave exciting force amplitude (N)

F_5 = pitch exciting moment amplitude (N.m)

g = gravitational acceleration ($m.s^{-2}$)

G = centre of gravity

GM = metacentric height (m)

i = imaginary root $\sqrt{-1}$

I = pitch moment of inertia ($kg.m^2$)

I" = added moment of inertia of the ship ($kg.m^2$)

Im = imaginary part of a complex number

k = deep water wave number (m^{-1})

K.n	=	spring stiffness of spring number "n" ($N.m^{-1}$)
K_{water}	=	spring constant of water
L	=	effective wavelength (m)
LDT	=	linear displacement transducer
LVDT	=	linear variable differential transformer
L.o.a	=	overall ship length (m)
L_{pp}	=	length of ship between perpendiculars (m)
L_{wl}	=	length of ship water line (m)
m"	=	sectional added mass coefficient per unit length along the ship ($kg.m^{-1}$)
M	=	longitudinal metacentre
M	=	mass of the ship (kg)
M"	=	added mass of the ship (kg)
N	=	sectional heave damping coefficient per unit length along the ship ($kg.s^{-1}.m^{-1}$)
O	=	centre of buoyancy
r	=	pitch radius of gyration (m)
R	=	restoring moment (N.m)
RAO	=	response amplitude operator
Re	=	Reynold's number
Re	=	real part of a complex number
S_w	=	sectional area under calm water line (m^2)
T	=	extreme draught (m)

1 INTRODUCTION

It has been found that the larger the vessel used to transport commodities, the less the cost per tonne of the commodity. This has led to a trend towards larger cargo carriers over the last two decades. The draught of these vessels has become so great that even areas in open sea, (eg. the North Sea), have to be regarded as restricted waters with respect to the ship's draught. Thus there is a need for the understanding of a ship's motions in shallow water, under wave action.

This knowledge is also required in the design of dredged approach channels with regard to grounding of the vessel. The influence of water depth on ship motions becomes noticeable when the water depth is less than four times the draught of the ship, (van Oortmerssen 1976). When this depth is reduced to less than twice the draught, the effect of the seabed is significant.

The motions of the ship are affected in two ways, firstly the incident waves are influenced by the proximity of the seabed, wave heights generally increasing with decreased depth. Secondly, the hydrodynamic coefficients of the ship vary dramatically with shallowness. Both these factors tend to increase the amplitudes of the ship's motions.

The dynamics of ship motions are governed by equations of motion which balance external forces and moments acting on the ship due to waves, with the restoring forces and moments due to gravity and inertia. Several methods have been proposed to solve these equations of motion for deep, restricted and shallow water cases. This thesis only considers the vertical motions in restricted and shallow water, for a Series 6Ø ship of block coefficient Ø,7, with no forward velocity. The relevant parameters and coefficients for the ship are found empirically and substituted into the equations of motion, which are then solved. The result is a motion amplitude for each water depth and water wave frequency. These are compared against results obtained purely from theory or experiment.

1.1 SHIP GEOMETRICAL PARAMETERS

Only dimensions and geometrical parameters relevant to this thesis are described here. For a complete overview of ship terminology, the reader is refered to Barnaby (1969), "Basic Naval Architecture" or similar texts quoted in the references.

The major ship dimension is obviously the ship's length. There are three common ways of measuring the length;

a) Length between perpendiculars....... L_{pp}

b) Length overall...................... L.o.a

c) Length of the water line............ L_{wl}

In dealing with the ship's vertical motions, the length between perpendiculars is normally used, the other two being more pertinent in drag and resistance calculations. The length between perpendiculars is measured parallel to the keel, between the fore and aft perpendiculars. The fore perpendicular is a vertical, drawn up from the most forward point on the summer load waterline. The aft perpendicular is a vertical, drawn up from the after side of the rudder post, or if the ship does not have such a post, from the centre line of the rudder pintals.

The midpoint of the length is called "amidships", and is normally the broadest section of the ship. The ship breadth "B" is measured at this point, in one of two ways. The "moulded breadth" is measured from the inside of the ship hull on one side, to the inside of the hull on the opposite side, at the water line. The "extreme breadth" is measured to the outside of the hull on either side at the water line.

The vertical parameter of a ship used for motion calculations is draught, "T". This is the vertical distance from the underside of the keel to the waterline. When the keel is parallel with the water line, the ship is said to be floating on an even keel. If the draught is greater at the bow, (or stern), then the ship is said to be "trimmed by the bow, (or stern)". The difference between the fore and aft draught is called the "trim" of the ship.

Draught, like breadth, can be measured either moulded or extreme. As this thesis is concerned with underkeel clearances, the extreme measurement is used throughout, without qualification.

The longitudinal metacentre is best explained with the aid of Figure 1.1. Consider a ship floating on an even keel and then displaced in a pitch mode as shown.

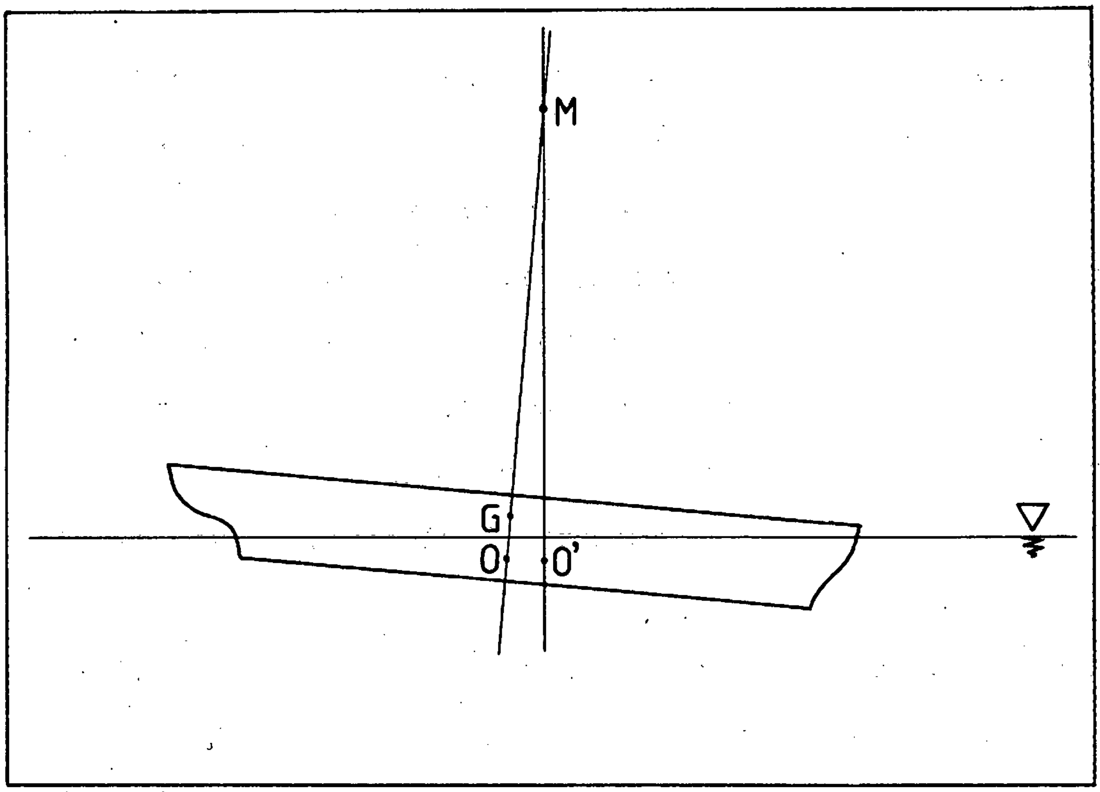


FIGURE 1.1

SHIP DISPLACED IN PITCH MODE

The original position of the centre of buoyancy, O, will move to a new position O'. If a line is drawn from O, perpendicular to the original waterline and another from O', perpendicular to the new waterline, the two lines will intersect at a point M. This point of intersection is called the "longitudinal metacentre" and the distance from the centre of gravity "G", to the longitudinal metacentre M, the metacentric height, "GM".

The "ship form" refers to the shape of the hull surface. As the curves of this surface are not generated from mathematical principles, it is necessary to have a numerical or graphical approximation to depict the shape. This is done via a "lines plan" which is a set of mutually orthogonal sections through the ship form. The lines plan for the model used in this thesis may be found in Appendix A.

The "block coefficient" is the ratio of the displaced volume of water "V", to its circumscribing rectangular block, given as:-

$$C_b = \frac{V}{L_{pp}.B.T}$$

Any other parameters will be defined as they occur in the text.

2 LITERATURE SURVEY

2.1 WATER WAVES

Water waves are produced by a number of actions, the most common being the action of wind on the air-water interface. The propagation of the wave once generated is controlled by physical properties of the water and environment. The speed at which the waveform travels is termed "celerity" and the distance between successive crests, (or troughs), the "wavelength". The "period" of a train of waves is the time elapsed between the passing of successive crests, viewed by a stationary observer.

2.1.1 WAVE CLASSIFICATION

Four major categories of waves can be distinguished;

a) Ripples

b) Waves of translation

c) Deep sea waves

d) Shallow water waves

All except ripples are "gravity waves", gravity being the major force involved. In the case of ripples, surface tension is the dominant force and the wave heights are very small compared to gravity waves. Waves of translation involve bodily movement of fluid in the direction of the motion of the wave and like ripples are of no consequence to this thesis.

Deep water waves are waves in which the water depth is large compared to the wavelength. It can be shown (Muckle 1975) that the seabed has no effect on wave motion if the water depth is greater than half the wavelength. Shallow water waves are those that occur in water of depth less than one twentieth of the wavelength. In this case the sea bed affects the movement of the water particles and hence the wave motion. It can be seen that there exists a class of waves between these two classes, partly dependent on water depth, termed "intermediate waves". The waves used in the restricted water depths of this thesis fall into the categories of shallow and intermediate waves.

2.1.2 WAVE THEORY

A number of different wave theories are available to predict the physical parameters of a wave. The classical theory is the "Airy theory" which is valid for waves of small wave height relative to the wavelength or water depth. This is because the theory assumes that the waves have a sinusoidal form and as the wave height increases, the wave form deviates from this sinusoidal shape. The general formulation giving the relationship between the wave celerity c, wavelength L and water depth d is:-

$$c = \sqrt{\frac{g.L}{2.\pi} . \tanh \frac{2.\pi.d}{L}}$$

This formula is derived by considering energy and total force principles within the wave. If the water depth is very large in comparison to the wavelength, the hyperbolic tangent term will tend to unity. Thus in deep water the celerity can be written as:-

$$c = \sqrt{\frac{g.L}{2.\pi}}$$

from which it is evident that the celerity of a deep water wave is dependent on the wavelength only. Note that the wavelength and wave period are always linked by

$$L = c.T_w$$

The hyperbolic tangent has the property that tanh(x) tends to x for small values of x, so that for small water depths, the equation reduces to

$$c = \sqrt{g.d}$$

Thus the speed of a shallow water wave is dependent on water depth only.

As this thesis deals with waves that fall into the categories of shallow as well as intermediate water waves, the full equation was used throughout. Note that "intermediate" is also referred to as "restricted" by some authors.

2.2 VIRTUAL MASS

2.2.1 THE CONCEPT OF ADDED MASS

When a body moves through a fluid, it causes a motion in the fluid around it, because it continually has to displace the fluid as it moves. This phenomena can also be visualised by considering the kinetic energy of the system. If a body moves with a steady rectilinear motion through an ideal fluid, which was originally at rest, the amount of kinetic energy of the system must remain constant.

However, if the body is now accelerated, the kinetic energy in the ideal fluid must also change. Extra energy has to be supplied as additional work is done on the fluid to increase its kinetic energy. The extra work is "felt" by the body as an effective additional drag, similar in form to the force required to accelerate the mass of the body. Hence it is convenient to visualise a mass of fluid as an "added mass" of fluid which is accelerated, or decelerated, with the body. No such mass can be distinguished in reality although the fluid around the body does assume a spatial velocity distribution, varying with time.

The sum of the mass of the ship and the added mass is known as the "virtual mass" of the body. As the added mass is dependent on various parameters, the virtual mass is not a constant value for a specific body. The added mass has been shown to vary with body shape, frequency of encounter of the body with wave motion of the fluid, and by far the most important factor, the proximity of boundaries in the fluid.

For a series 60 ship hull, it has been found that the virtual mass in deep water is approximately twice the mass of the displaced water, whereas at a depth to draught ratio of 1,1, the virtual mass increases to over four times the mass of the displaced water. In the case of a floating body, the mass of the displaced water is equal to the mass of the body.

2.2.2 COMPUTATION OF A SHIP'S ADDED MASS

Various methods have been used to calculate the added mass of a ship for given values of parameters, by empirical means. From a study of such empirical data, Todd (1961) found a formula for the virtual mass of a ship in deep water:-

$$\text{VIRTUAL MASS} = M.\left(1,2 + \frac{B}{3.T} \right)$$

where M = mass of the ship

T = draught of the ship

B = breadth of the ship

For the ship under study in this thesis, this formula gives the virtual mass as 2,Ø3.M in deep water.

It can be shown that the added mass of a cylinder floating in deep water, with its diameter on the calm water line, is equal to the mass of the displaced water, ie:-

$$\text{ADDED MASS} = \frac{\pi.\rho.a^2}{2} \quad \text{per unit length}$$

where a = radius of the cylinder

Based on this knowledge, Muckle (1975) suggested an approximate method of finding the added mass of a ship form in deep water:-

$$\text{ADDED MASS} = \int \rho . S_w . \frac{y_w}{T_m} . dx$$

where ρ = density of water

T_m = mean draught of section

y_w = half breadth of cross-section on calm water line

S_w = area of ship cross-section below the calm water line

This formula has no dependency on the water depth and is therefore a constant for the ship form. The above is only concerned with acceleration through originally still water, ie not waved water. Ursell (1962) gives a formula for the added mass of a circular cylinder in waved water, valid for short period waves, but does not include a water depth parameter:-

$$\text{ADDED MASS} = M . \left(1 - \left(\frac{4}{3} . \pi . w^2 . \frac{a}{g} \right) + O\left[\frac{g}{w^2 . a} \right] \right)$$

where w = frequency of encounter

The effect of the waved water is to vary the displaced volume of water and thereby alter the resulting added mass. Thus, according to this formula, for waved water the added mass is less than that for non-waved water.

Kim (1968) brought the influence of water depth into his equation for added mass as a correction factor applied to the added mass for deep water.

$$m''_{d=d} = m''_{deep} + \rho.\frac{B^2}{\pi}.(G_{2s+1}(k.d)_{s=\emptyset})$$

and $$m''_{deep} = \rho.\frac{B^2}{\pi}.(\ -\ln(\emptyset,89\emptyset.k.B)\ )$$

where m'' = added mass per metre of ship

d = water depth

k = deep water wave number = $\frac{2.\pi}{L}$

From this formula it can be seen that the influence of the shallow water on the magnitude of added mass is controlled by the singular function:-

$$G_{2s+1}(k.d)_{s=\emptyset}$$

Note that this function is only valid for the deep water wave number tending to zero, ie long period waves. For the short period range, Havelock (1958) gives a formula for added mass in restricted water depths:-

$$m''_{d=d} = m''_{deep} \cdot \rho \cdot \frac{\pi}{8} \cdot B^2 \cdot (1 + 0,5 \cdot (\frac{T}{d})^2)$$

These semi-empirical methods have now been superseded by more detailed analytical methods. As early as 1929, Lewis suggested a "conformal transformation" method, whereby he derived a series of prismatic sections from a cylinder. Assuming a two-dimensional flow, he transformed the fluid flow around the cylinder to that around the new section to calculate the energy of the entrained water.

There exists a variety of numerical techniques for the calculation of added mass and damping coefficients. Korvin-Kroukovsky and Jacobs (1957), Salvensen, Tuck and Faltinsen (1970) and Ogilvie and Tuck (1969) have done extensive work on the problem. Korvin-Kroukovsky and Jacobs used the strip theory, solving the two dimensional boundary value problem for each cross section of the ship. The two dimensional-solutions were then adjusted to include certain three dimensional and forward speed effects, based on intuitive physical arguments. Salvensen et al started out with a three dimensional boundary value problem and made simplifying assumptions that the ship was slender, the motions small and frequency high.

Ogilvie and Tuck followed the same approach, but made the simplifications in a more consistent manner, introducing a motion amplitude parameter to express the smallness of the motion and a slenderness parameter as a measure of the transverse parameters of the ship. Experimental agreement appears to be better for the Ogilvie and Tuck method over the other two.

Other methods of solving the problem include finite-element variational formulations by Bai and Yeung (1974) and Chen and Mei (1974). More recently, Berhault (198Ø) used an integro-variational method to obtain very good results. These elaborate procedures are however, normally too time consuming to allow one to investigate efficiently the effects of various parameters.

Recent work by Yeung (198Ø), following the formulation described by Garrett (1971), solved the hydrodynamic problem by matching the eigen-functions of the interior and exterior problems. The resulting infinite system can be solved directly. This appears to be one of the most time effective numerical techniques available to date.

2.3 DEFINITION OF SHIP MOTIONS

2.3.1 CO-ORDINATE SYSTEM

The "right hand" co-ordinate system is used throughout this thesis as shown schematically below.

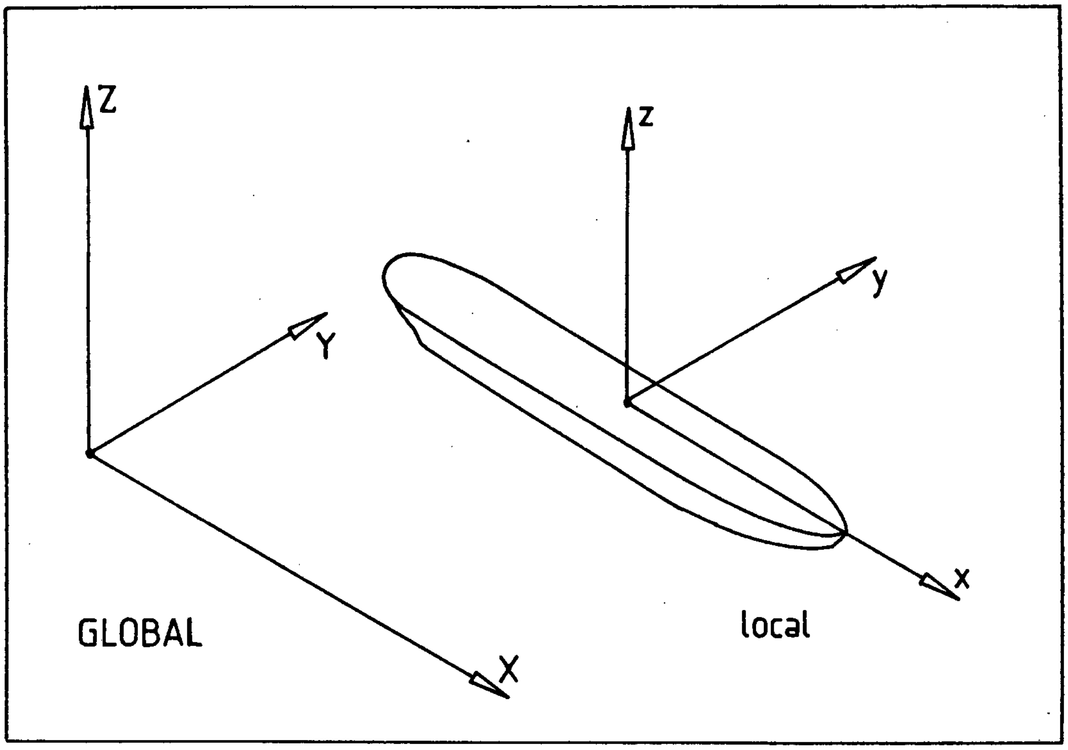


FIGURE 2.3

CO-ORDINATE SYSTEM

The global X-axis lies on the undisturbed water surface, parallel to the undisturbed central ship axis, the Z-axis is positive vertically upwards and the Y-axis is positive to port also on the undisturbed water surface. Locally the origin is situated at the centre of gravity of the ship, with the x-axis parallel to the ship's water line and the central axis of the ship. The z-axis is positive upwards, normal to the ship's water line and therefore vertical when the ship is floating on an even keel. Again the y-axis is positive to port, orthogonal to the other two axes. If the centre of gravity is taken to be at the same level as the water line, (as it is in this thesis), then when the ship is at rest, floating on calm water, the local and global co-ordinate systems coincide.

2.3.2 SHIP MOTIONS

A ship floating on a disturbed water surface is able to move in two degrees of freedom, (translational and rotational), in and about each axial direction. Thus a total of six possible modes of motion can be defined. None of the six modes are necessarily exclusive of any other and it will be shown later, that due to coupling, each mode is mutually inclusive of at least one other mode of motion. The six possible motions are shown in figure 2.3.2

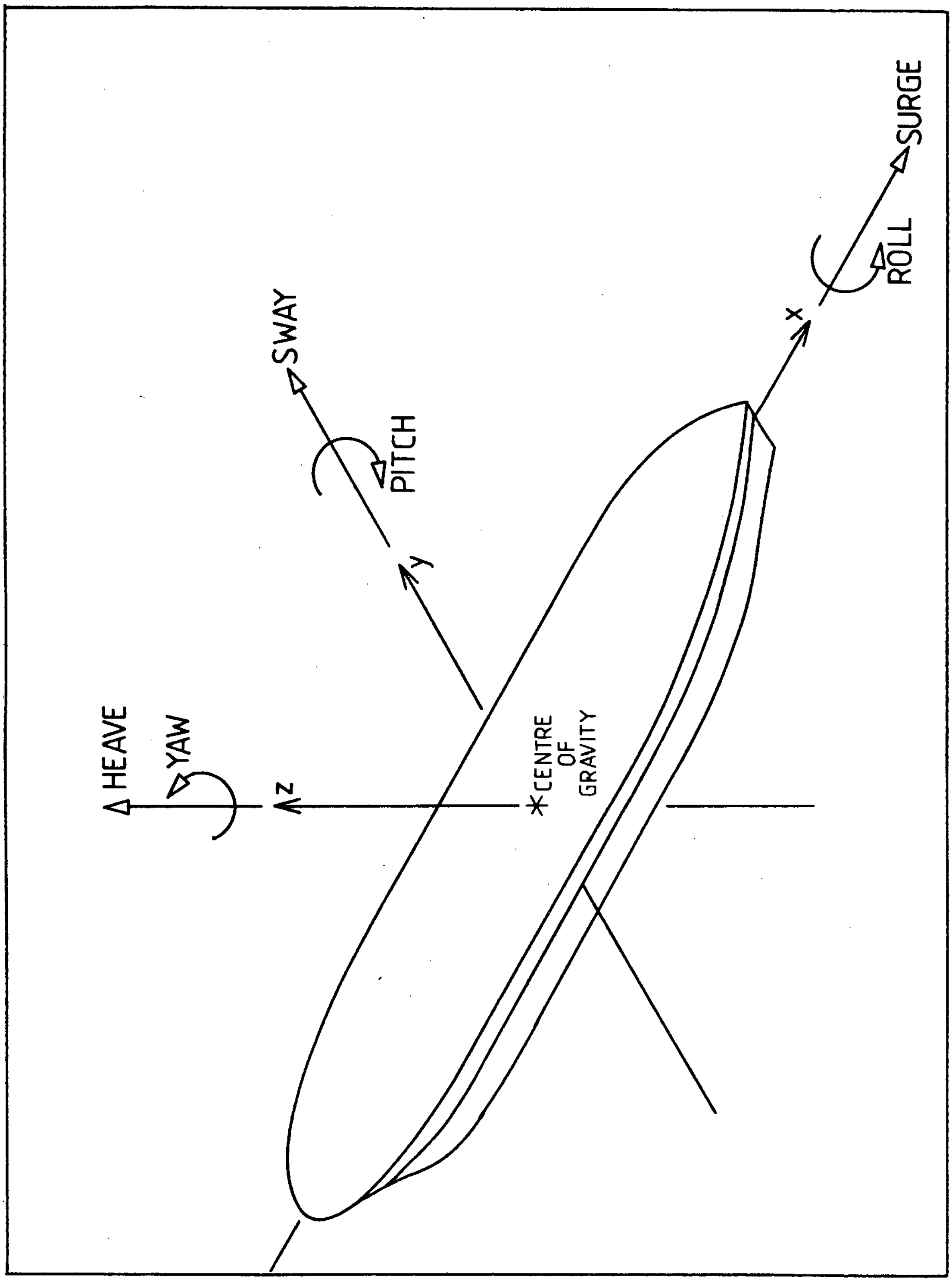


FIGURE 2.3.2
MODES OF OSCILLATORY MOTION

2.3.3 PURE MOTION EQUATIONS

Before any calculations can be made on the motions of a ship in a seaway, certain simplifying assumptions have to be made, the standard assumption being that the fluid is ideal, that is, it is inviscid, incompressible and homogeneous. It is also assumed that the fluid's motion is irrotational and the only significant body force acting on it is gravity.

The waves acting on the system are considered to be regular, sinusoidal, long crested and of small amplitude. Unlike the fluid assumptions which are reasonable close to reality, the assumed wave action is not representative of that found at sea. In a storm, waves of different heights, lengths and directions are generated which all combine to give the sea its "confused" state. It is only at some distance from the storm that any appearance of regularity might be noticeable due to the faster celerity waves outrunning the slower ones. As it is impractical to analyse all possible sea conditions, it is usual practise to use only the significant wave height and period.

The ship itself is normally taken as rigid longitudinally, although some authors have recently moved away from this, e.g. Bishop, Price and Tam (1977) who treat the hull as a Timoshenko beam. Finally, with respect to the actual motions, an assumption of linearity is taken, such that doubling the wave amplitude doubles the response. This is a valid assumption provided the motions are small relative to the transverse dimensions of the ship.

The dynamics of ship motions are governed by equations of motion which balance external forces acting on the ship, with the restoring forces due to gravity and inertia. Note that the word "force" is used in a general sense, taken to include both moment and true force components acting on the ship.

Assuming the ship to be in stable equilibrium in calm water, its weight is balanced by the hydrostatic pressure on the hull. These forces may be discounted by considering only the unsteady perturbations. The principal unsteady forces acting on a free floating ship are due to hydrostatic and hydrodynamic components of the pressure acting on the hull. Additional forces may come from aerodynamic forces on the superstructure and the forces from the propeller(s). These additional forces are normally neglected for simplicity as their effect on the motion response is minimal.

2.3.3.1 PURE HEAVE MOTION

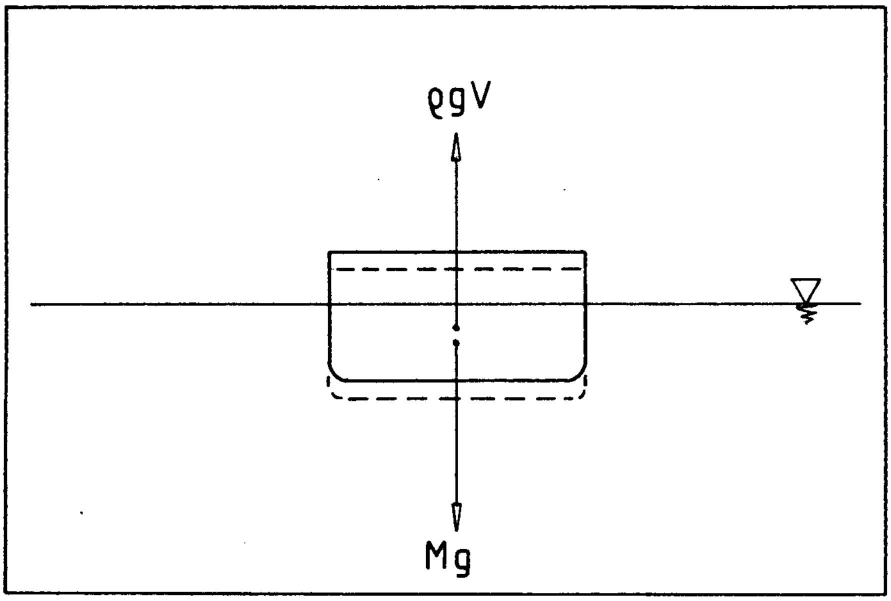


FIGURE 2.3.3.1

SHIP IN HEAVE MODE

Consider a ship heaving in otherwise calm water. Figure 2.3.3.1 shows a ship in its equilibrium position, where its weight Mg is balanced by the buoyancy ρgV. The dotted line shows the ship instantaneously heaved into the water by an amount z downwards. The increase in buoyancy, assuming that the waterplane area A remains unchanged, will be ρgAz and this is the restoring force. The ship will have an acceleration $\ddot{z}$ so that, ignoring damping effects, the equation of motion is:-

$$(M+M'').\ddot{z} = -\rho.g.A.z$$

where M" = added mass

From the solution of this equation, the theoretical natural undamped period of oscillation will be:-

$$T_h' = 2.\pi\sqrt{\frac{a}{c}} = 2.\pi\sqrt{\frac{M+M''}{\rho g A}}$$

This does not represent a true motion however, as all oscillating problems involving a viscous liquid are subject to viscous damping effects. If we assume viscous damping is proportional to the velocity "$\dot{z}$" through a damping factor "b" and represent the restoring force by "cz", then the unforced equation of motion for free oscillation becomes:-

$$(M+M'').\ddot{z} + b.\dot{z} + c.z = 0$$

When a ship moves through regular long waves, an additional vertical force acts on the hull, due to the position of the centre of buoyancy oscillating vertically in sympathy with the moving water surface. This force can be shown to be a simple harmonic function of time. The effective frequency at which the ship "sees" the wave, taking into account the ship's velocity and the possible obliqueness of the waves on the ship, is called the "frequency of encounter".

Froude formulated an equation of motion for a forced single mode motion due to wave action, based on the following assumptions:-

1) Waves are regular and sinusoidal.
2) The ship encounters the wave at constant speed.
3) Energy interchange on the hull between the vessel and the waves is neglected.
4) Motion is isochronous.

The equation of motion for a forced single degree of freedom system can then be represented as:-

$$a.\ddot{z} + b.\dot{z} + c.z = F.\cos(w.t + e_h)$$

where $a = (M + M'')$

F = amplitude of exciting force

w = frequency of encounter

e_h = phase difference between wave and exciting force

The magnitude of the resulting heaving motion is dependent on several factors, but reaches a maximum when the frequency of encounter equals the natural heaving frequency of the vessel and resonance occurs.

Assuming that away from this condition, the heave magnitude is given by:-

$$z = z_o . \sin(w.t - \phi)$$

then from vibration analysis, the solution of the eqation is:-

Amplitude of heave, $z_o = \dfrac{F}{\sqrt{(b.w)^2 + (c-a.w^2)^2}}$

Phase angle, $\tan \phi = \dfrac{b.w}{(c - a.w^2)}$

Damped period of oscillation:-

$$T_h = \frac{2.\pi}{\sqrt{\frac{c}{a} - \frac{b^2}{4.a^2}}}$$

Thus if the ship is subjected to resonance conditions, it is only possible to avoid amplified heave motions by altering the velocity or heading angle of the ship, as the other variables cannot be altered easily.

2.3.3.2 PURE PITCH MOTION

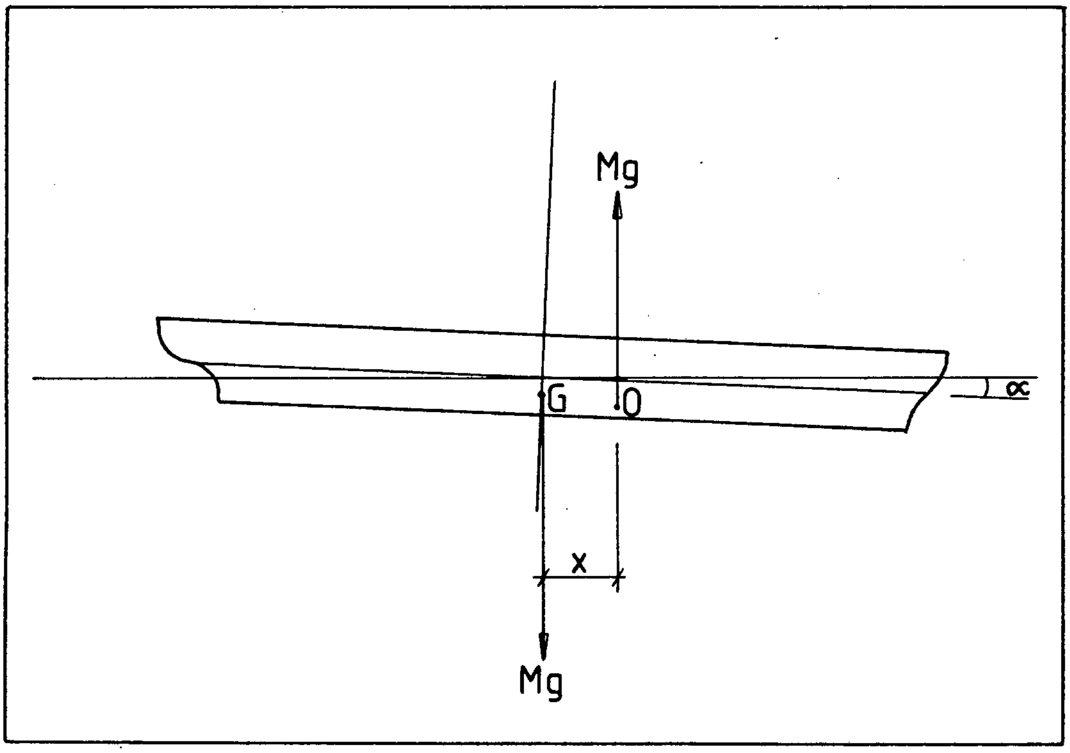


FIGURE 2.3.3.2

SHIP IN PITCH MODE

Consider a ship pitched at an angle α, as shown in Figure 2.3.3.2. The moment tending to restore the ship to its equilibrium position is Mgx. Given that the moment of inertia of the ship in pitch is "I" and the angular acceleration is "$\ddot{\alpha}$", then

$$(I+I'').\ddot{\alpha} = -M.g.x$$

The lever arm, "x" is a function of the angle of pitch "α" and if we restrict motion to small angles, is given as:-

$$x = GM.\alpha$$

where GM = the metacentric height

Substituting this into the equation and assuming that viscous damping is proportional to the angular velocity, through a damping factor "b", the unforced equation of motion becomes:-

$$(I+I'').\ddot{\alpha} + b.\dot{\alpha} + M.g.GM.\alpha = 0$$

As for the case of heave, the effect of regular long waves on the ship causes a moment which can be shown to be a simple harmonic function of time. This is due to the fact that the wave action causes the centre of buoyancy to move horizontally. If the exciting wave motion is sinusoidal, then the resulting centre of buoyancy movement will also be sinusoidal and can be represented by:-

$$\text{Axial motion of } O = O_a.\sin(w.t)$$

where O_a = amplitude of movement

This is then the forcing function of the motion causing the pitching motion and can be generalised in the form:-

Exciting Force $= F.\cos(w.t + e_P)$

where F = amplitude of exciting force

w = frequency of encounter

e_P = phase difference between wave and exciting moment

Thus the forced equation of motion is:-

$$a.\ddot{\alpha} + b.\dot{\alpha} + c.\alpha = F.\cos(w.t + e_P)$$

where $a = (I+I'')$

b = damping coefficient

$c = M.g.GM$

From vibration analysis, it can be shown that the solution to the equation of motion is:-

$$\text{Amplitude of pitch,} \quad = \quad \frac{F}{\sqrt{(b.w)^2 + (c-a.w^2)^2}}$$

$$\text{Phase angle,} \quad \tan\phi = \frac{b.w}{(\,c - a.w^2)}$$

Natural damped period of oscillation:-

$$T_P = \frac{2.\pi}{\sqrt{\frac{c}{a} - \frac{b^2}{4.a^2}}}$$

Ignoring the viscous damping, the theoretical natural undamped period of oscillation will be:-

$$T'_p = 2.\pi\sqrt{\frac{a''}{c''}} = 2.\pi\sqrt{\frac{I+I''}{M.g.GM}}$$

As for the case of heave, resonance occurs when the frequency of encounter equals the natural frequency of the pitching motion. The ship designer has little control over this natural period as GM is of the order of the ship's length and hence any small changes made to the height of the centre of gravity will have negligible effect.

2.3.4 THE COUPLED EQUATIONS OF MOTION

From Newton's law and for a ship of lateral symmetry, the equations of motion for the six degrees of freedom are found to consist of two sets of coupled linear differential equations. One set deals with the vertical motions and includes surge, heave and pitch, whilst the other deals with lateral motions and includes roll, sway and yaw. Only the former group are looked at in this thesis. If it is assumed that the ship has a long and slender hull form as well as lateral symmetry, then it can be shown (Salvensen, Tuck and Faltinsen 197Ø), that the hydrodynamic forces associated with surge motions are much smaller than forces associated with heave and pitch. Hence surge coupling can be ignored leaving only coupling effects between heave and pitch motions.

To differentiate between coefficients, the subscript "3" for heave and "5" for pitch, are used in the form "jk", where j represents the mode of motion being considered due to the forcing mode k. The equations of motion for a ship heaving and pitching in regular waves are then:-

$$\left[A_{33}.\ddot{z} + B_{33}.\dot{z} + C_{33}.z\right] - \left[A_{35}.\ddot{\alpha} + B_{35}.\dot{\alpha} + C_{35}.\alpha\right] = F_3.\cos(w.t + e_3)$$

$$\left[A_{55}.\ddot{\alpha} + B_{55}.\dot{\alpha} + C_{55}.\alpha\right] - \left[A_{53}.\ddot{z} + B_{53}.\dot{z} + C_{53}.z\right] = F_5.\cos(w.t + e_5)$$

Where :-

A_{33} = Virtual mass in forced heaving (kg)

A_{55} = Virtual inertia in forced pitch ($kg.m^2$)

A_{35} , A_{53} = Coupling coefficients linking the two modes of motion (kg.m)

B_{33} = Heave damping coefficient ($kg.s^{-1}$)

B_{55} = Pitch damping coefficient ($kg.m^2.s^{-1}$)

B_{35} , B_{53} = Coupling damping coefficients linking the two modes of motion ($kg.m.s^{-1}$)

C_{33} = Heave hydrostatic restoring force ($kg.s^{-2}$)

C_{55} = Pitch hydrostatic restoring force (N.m)

C_{35} , C_{53} = Coupling restoring force coefficients linking the two modes of motion (N)

F_3 = Heave exciting force amplitude (N)

F_5 = Pitch exciting moment amplitude (N.m)

The hydrodynamic coefficients are derived from the strip theory, (Kim 1968) and given as :-

$$A_{33} = \int m''.dx + M$$

$$B_{33} = \int N.dx$$

$$C_{33} = 2.\rho.g.\int y_w.dx$$

$$A_{35} = \int m''.x.dx + \rho.\int S_w.x.dx$$

$$B_{35} = \int N.x.dx - U.\int m''.dx$$

$$C_{35} = 2.\varrho.g.\int y_w.x.dx - U.\int m''.dx$$

$$A_{55} = \int m''.x^2.dx + I$$

$$B_{55} = \int N.x^2.dx$$

$$C_{55} = 2.\varrho.g.\int y_w.x^2.dx - U.\int N.x.dx - U^2.\int m''.dx$$

$$A_{53} = \int m''.x.dx + \varrho.\int S_w.x.dx$$

$$B_{53} = \int N.x.dx + U.\int m''.dx$$

$$C_{53} = 2.\varrho.g.\int y_w.x.dx$$

where ϱ = water density

S_w = sectional area under calm water level

y_w = half breadth of section on calm water line

m'' = sectional added mass coefficient per unit length of ship

N = sectional heave damping coefficient per unit length of ship

M = mass of ship

I = longitudinal moment of inertia

U = forward velocity of ship

All integrals are taken over the length of the ship

As the oscillating system is considered to be linear and harmonic, the motions of the system must also be linear and harmonic and hence the motions may be represented as the real part of:-

$$z = z_o . e^{-i.w_e t}$$

and

$$\alpha = \alpha_o . e^{-i.w_e t}$$

where

i represents the imaginary root $\sqrt{-1}$

and z_o, α_o are motion amplitudes

The solution of the equations of motion, to obtain the motion amplitudes and phase angles, is discussed in section 3.1.

2.3.5 SOLUTION OF THE EQUATIONS OF MOTION

Examination of the coefficients of the equations of motion, given in 2.3.4, shows that all can be evaluated by numerical integration over the length of the ship, providing one knows the two dimensional distribution of added mass and damping coefficients along the ship. The computation of these two parameters is the most time consuming part of computing the ship motions.

For the deep water case, Korvin-Kroukovsky and Jacobs (1957) proposed a means of calculating the required parameters using a "Strip Method". This theory uses the fact that the length of the ship is large relative to the lateral and vertical dimensions, thus the three dimensional problem can be reduced to a two dimensional problem. A restriction of the theory is that the wavelength must be short, ie of the order of the breadth of the ship, so that no interaction occurs between sections. This method was the first to give acceptably accurate results for engineering applications. However, at the time it was felt that the theory was not derived in a rational mathematical manner, but rather by the use of "physical intuition". A major objection to the theory is that the forward speed terms in the equations of motion did not satisfy the symmetry relationship proved by Timman and Newman (1962). Considerable work has been done on the theory since then, notably Grim (1960), Kim (1968) and Ogilvie and Tuck (1969).

To take into account the problem of limited depth, Wilson (1958) empirically increased the hydrodynamic mass and damping of the ship and Wendel (1950) found the theoretical hydrodynamic mass by conformal mapping. Andersen (1979) used finite element techniques to solve the problem numerically, but computations require an increasingly large number of integration points as the water depth decreases.

Kim (1968) successfully adapted the strip theory for a restricted water depth, valid for vertical modes of motion only. It cannot be used for lateral motions since the strip theory is based on a two dimensional model, requiring that the flow of water passes entirely longitudinally underneath the keel of the ship. In shallow water however, three dimensional effects become important as the water flows around the ends as well as under the keel. In the extreme case of a ship sitting on the seabed, water can only flow around the ends.

A major contribution to the shallow water case was made by Tuck (1970), Tuck and Taylor (1970) and Beck and Tuck (1971). They derived a "slender body" theory based on three dimensional effects, solving the problem by a method of matched asymptotic expansions. This theory is applicable when the wavelengths are of the order of the ship's length. Plotkin (1977) expanded the theory to include the effects of forward speed of the ship. The high frequency waves however cause the added mass coefficient (m") to approach zero and the damping coefficient (N) to tend towards a constant. These results are unacceptable and under such circumstances, the strip theory for moderate depths of Kim (1968) is more accurate. Thus for long period waves the slender body theory is more suitable and for short periods, the strip theory. On comparison of results, it can be seen that for the long period waves, where the slender body theory is more accurate, the strip theory tends to under-estimate the ship motions. Similarly, for short period waves, the slender body theory under-estimates the ship motions.

A solution method that is not restricted by frequency regimes is provided by the three dimensional source technique. Originally used for computing wave loads on offshore structures by Daubert (1970), Garrison and

Chow (1972) and van Oortmerssen (1972), it was adapted by Yeung (1973) for ship shaped bodies. Due to the large number of elements required to schematize the entire boundary surface, resulting in an even larger number of equations, the method is impractical from an engineering standpoint.

Newman (1978) showed that as long as the assumption of small amplitude unsteady motions is valid, linear superposition can be applied. Thus it is possible to consider separately the "radiation problem", where the ship undergoes prescribed oscillatory motions in otherwise calm water, and the "diffraction problem", where incident waves act on the ship. Interactions between these two first order problems are of second order and therefore neglected in linear theory. This approach was also used by Tasai, Takaki and Ohkusu (1978) for their empirical investigations.

It can therefore be concluded that although three dimensional approaches have been developed for the prediction of ship motions, they are still too time-consuming for a designer's purpose. Thus the strip theory is normally used for deep and restricted water depths and other methods are only used for the shallow water case.

3 COMPUTER PROGRAMS

Although some computer programs already exist for the analysis of ship motions in restricted water depths, these were not suitable for the analysis techniques required in this thesis. In this instance it was required that all the coefficients of the equations of motion be entered as input data and not calculated within the program. Thus, what was required was a program that would accept all the relevant data, set up the equations of motion and solve them. As stated in the terms of reference of this thesis, only the coupled motions of heave and pitch would be investigated.

3.1 SOLUTION OF THE EQUATIONS OF MOTION

The equations of coupled motion as given in 2.3.4 are:-

$$\left[A_{33}.\ddot{z} + B_{33}.\dot{z} + C_{33}.z\right] - \left[A_{35}.\ddot{\alpha} + B_{35}.\dot{\alpha} + C_{35}.\alpha\right] = F_3.\cos(w.t + e_3)$$

$$\left[A_{55}.\ddot{\alpha} + B_{55}.\dot{\alpha} + C_{55}.\alpha\right] - \left[A_{53}.\ddot{z} + B_{53}.\dot{z} + C_{53}.z\right] = F_5.\cos(w.t + e_5)$$

As the oscillating system is considered to be linear and harmonic, the motions of the system must also be linear and harmonic and hence the motions may be represented as:-

$$z = z_o . e^{-i.w_e t}$$

and

$$\alpha = \alpha_o . e^{-i.w_e t}$$

where

i represents the imaginary root $\sqrt{-1}$

and z_o , α_o are the motion amplitudes

Thus it can be shown that:-

$$\dot{z} = -i.w.z_o . e^{-i.w_e t}$$

$$\ddot{z} = -w^2 . z_o . e^{-i.w_e t}$$

and similarly for pitch.

Hence the coupled equations can be represented in the complex matrix form:-

$$(P + i.Q).H = F$$

$$\text{where } P_{jk} = -w^2 . A_{jk} + C_{jk} \quad \text{(real)}$$

$$Q_{jk} = -w.B_{jk} \quad \text{(real)}$$

giving a real solution in the form:-

$$H_{ko} = \sqrt{(Re(H_k)^2 + Im(H_k)^2)}$$

and the phase angle $\phi_k = \tan^{-1}(Im(H_k) / Re(H_k))$

where Re = the real part of...

Im = the imaginary part of...

As a self check, the matrix was solved by two different methods, one by direct multiplication of the matrix components, and the other by matrix manipulation techniques. The direct method required less computer time and was therefore used in the final program. The alternative method is also based on the general solution of a 2x2 complex matrix, but it does not involve any complex number manipulations. The solution is given as:-

$$H = (P + i.Q)^{-1}.F$$

where $(P + i.Q)^{-1} = C + i.D$

$$C = ((P+Q)^{-1} + (P-Q)^{-1}).R$$

$$D = ((P+Q)^{-1} - (P-Q)^{-1}).R$$

$$R = ((P+Q).(P-Q)^{-1} + (P-Q).(P+Q)^{-1})^{-1}$$

3.2 PROGRAM DESCRIPTION - "SMISW"

A brief description of the program "SMISW", (Ship Motions In Shallow Water), is given here, a listing thereof can be found in Appendix B.

The program accepts details of the ship's geometrical properties and parameters for a given number of cross-sections of the vessel. Data specific to each particular run is then entered, such as wave frequency and water depth. All of this input data is read from data files, which have been set up using the auxillary program "SHIP".

The frequency of encounter "w", is calculated using:-

$$w = (w_{wave} + v_o.U)$$

where w_{wave} = wave frequency

U = ship velocity

v_o = shallow water wave number

The shallow water wave number is found from the equation:-

$$\frac{w^2}{g} = v_o.\tanh(v_o.d)$$

where d = water depth

The program then calculates the associated period of encounter and effective wave length from the formula:-

$$\text{PERIOD} = \frac{2.\pi}{w}$$

and

$$\frac{L}{T_w} = \sqrt{\frac{g.L}{2.\pi} .\tanh \frac{2.\pi.d}{L}}$$

where L = effective wave length

T_w = wave period

The formula for wave length is that given by the Airy theory, for waves in intermediate water depths and of small wave heights. Note that the calculation of the wavelength, like the shallow water wave number, is iterative.

Having calculated all the parameters and constants required for the equations of motion, the program reads in the coefficients found empirically.

With this data it is now possible to set up the left hand side of the equations on p.39, but the exciting forces require further calculation. The reason for this is explained in chapter 6. The exciting force equations as given by strip theory (Kim, 1968) are:-

$$F_3 \cdot \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix} = 2.\rho.g.a.\int y_w.K_1.dx - w^2.a.\int m''.K_1.dx$$
$$\pm\ w.a.\int N.K_2.dx$$

$$F_5 \cdot \begin{bmatrix} \cos\sigma \\ \sin\sigma \end{bmatrix} = -2.\rho.g.a.\int y_w.x.K_1.dx \pm w\ .a.\int N.x.K_2.dx$$
$$\mp\ w.U.a.\int \frac{dm''}{dx}.K_2.dx + w^2.a.\int m''.x.K_1.dx$$

where

$$K_1 = \frac{\cosh(\ v_o.(\ d-T_m\ )\ )}{\cosh(\ v_o.d\ )} \cdot \begin{bmatrix} \cos(\ v_o.x\ ) \\ \sin(\ v_o.x\ ) \end{bmatrix}$$

$$K_2 = \frac{\cosh(\ v_o.(\ d-T_m\ )\ )}{\cosh(\ v_o.d\ )} \cdot \begin{bmatrix} \sin(\ v_o.x\ ) \\ \cos(\ v_o.x\ ) \end{bmatrix}$$

and T_m = the mean draught of a section

Using the coefficients found empirically and assuming longitudinal distributions as described in chapter 5 for the added mass and damping coefficient, the exciting forces are computed. The equations of motion are now complete and are put into matrix form, as described in 3.1.

The matrix is solved for the amplitudes of motion and relative phase angles. This data can be printed out and/or stored on disc for use in the graphical plot program, "GRAPH".

3.2.1 GRAPHICAL PLOTTING PROGRAM - "GRAPH"

An auxiliary program has been written to handle graphical presentation of the previously calculated data. This program has the ability to accept any number of data files which will then be plotted on the same graph for comparison. Separate graphs are used for each mode of motion.

The graphical presentation can be either on the screen, or on a plotter. The graphical results given in chapter 6 have been produced by the latter method.

3.3 VERIFICATION OF THE PROGRAM

As a final check that the program was producing the required data, it was run using data published by Kim (1968). The coefficients were calculated using the strip theory and added masses and damping coefficients taken from published findings. The resulting motion amplitudes agreed excellently with the corresponding published results. Hence the program was presumed to be mathematically correct.

4 EXPERIMENTAL WORK

4.1 EXISTING FACILITIES

4.1.1 WAVE BASIN

It was essential for pure motion tests that the ship induced waves did not reflect off boundary walls and interact with the oscillatory motion of the model. For these tests the 5x5 m wave basin was used. Originally the basin had a beach around its perimeter of 12 mm granite chips to a height of 175 mm and slope of 1:2. Using this beach system to absorb waves would restrict the maximum water depth to less then 170 mm. As the required maximum for this thesis was 285,8 mm, (to achieve a depth to draught ratio of 2,5), the beach system had to be changed. To increase the size of the beach would reduce the effective water area of the basin which was undesirable. Following tests on different types of wave absorbers, and their relative efficiency, by Paterson (private communication), it was decided to replace the beach with vertical sheets of foam rubber. The sheets were mounted on wooden frames which were bolted 50 mm away from the basin wall, to form a continuous inner wall around the basin. The efficiency of wave absorption of the foam was just over 50% that of the beach for a 50 mm wave height and deteriorated with increased wave height.

4.1.2 WAVE CHANNEL

For the wave exciting forces it was necessary to have a source of stable sinusoidal waves, and for these tests, the wave channel was used. The channel is 91,4 m long by 1,524 m wide. A paddle type wave generator, capable of forming regular waves of various periods and amplitudes, is situated at the north end. At the south end, the waves break on to a beach of 25 mm granite chips, sloped at 1:20. Halfway between the wave generator and the beach is an underground viewing room. This overlooks the test section of the channel, where the floor has been elevated and accurately levelled.

The channel has the facility for a variable speed towing trolley to run along its length on rails. The original vertical motion tests by Blumberg made use of this trolley, but it was not required for this thesis.

4.2 DATA RECORDING

4.2.1 MEASUREMENT OF VERTICAL DISPLACEMENTS

A method of monitoring the model's vertical motions as a function of time had to be devised. Several different methods were considered, the major ones being, twin wire probes, slide wire potentiometers, linear variable differential transformers (LVDT), and linear displacement transducers (LDT). Consisting of a rod moving in a coil, the LVDT was the only method of measurement that exerted no form of friction or force on the model, and its excellent accuracy made it the obvious choice. However the device proved to be expensive and none were presently available in the country, meaning a minimum delay of six weeks for import. Twin wire probes have been used successfully for wave height records, but for this situation it was thought that the accuracy achieved on the rapid oscillations of the model would be reduced. Slide wire potentiometers provided an accurate form of measurement, but it was not easy to incorporate the frictional resistance of the device into the equations of motion.

The final solution therefore was to use the linear displacement transducers. These consist of a stainless steel rod mounted in a cylinder with teflon bearings. By supporting the cylindrical casing the rod could move vertically with the minimum of friction. This rod was then fixed via a spring to a 10 mm sprung steel cantilever, rigidly attached to the casing. The cantilever had a strain gauge bridge attached to it, such that, given an input voltage, the resulting output voltage varied linearly with displacement. As it was planned to use springs to enhance pure motion tests, (see section 4.4), the extra springs from the LDT's presented no problem.

Two LDT's were used for the pitch motions both had a maximum travel of 100 mm. For heave motion measurement, only one LDT was necessary and although one of the above could have been used, it was decided to use another which incorporated a dial gauge. This therefore had the added advantage of giving an instant visual indication of the model's motions and had a travel of 20 mm.

All three LDT's were calibrated by applying a 10 V input voltage and then displacing the LDT by a known amount, measured with vernier calipers. The range of each calibration was kept to 20 mm, as this was the expected maximum vertical motion to be measured during tests. By monitoring the output voltage, and fitting a "best fit line" through the points, a constant for each LDT could be found in the form of x millivolts per millimetre displacement.

4.2.2 MEASUREMENT OF EXCITING FORCES

In the exciting force tests, it was a force that had to be measured instead of a displacement. The ship was held rigidly by two cantilevers with adjustable vertical supports at the ends. (See section 4.7). Close to the point of support of the cantilever, 50 mm square areas were machined flat giving a fractionally thinner cross section than the rest of the cantilever. This had the effect of concentrating, to a degree, the stresses in the cantilever to this point. Strain gauge bridges were glued to these areas, such that the strain at these points could be measured as before. Assuming that the strain is directly proportional to the stress, and that we remain within the elastic range of the steel, a direct measurement of force is possible.

The cantilevers were calibrated for both positive and negative forces. By fitting a straight line to the points obtained, using the "least squares method", a constant for each cantilever was found in the form, x millivolts per newton force.

4.2.3 POWER SUPPLY

To supply the strain gauge bridges with a basic voltage input, a reliable voltage source was required. For this purpose, a power supply module was used. It had the facility to give a voltage supply of between zero and twelve volts, set by coarse and fine setting dials. Using the datalogger as a voltmeter, (see section 4.2.4), it was possible to set the voltage accurately to within Ø,1 millivolts. Once warmed up, the module would give a steady voltage output which however drifted from the set voltage by about Ø,1 mV per hour.

4.2.4 DATA AQUISITION UNIT

To read and record successive voltage readings from the outputs of the strain gauge bridges, a data aquisition/control unit was used, (referred to throughout this thesis as the datalogger). This unit was capable of taking and storing up to sixty successive readings in a minimum period of one second. More readings could be taken, but this involved transferring stored readings to the computer which caused undesirable time delays. The datalogger was capable of reading voltages down to one microvolt, but due to stray static sources, readings were only deemed accurate to Ø,1 mV.

4.2.5 MICRO-COMPUTERS

The datalogger was controlled by an HP Series 15Ø touch screen personal computer. The computer had to instruct the datalogger when to take readings, receive the readings once they were completed and convert them to displacements or forces for further analysis. Due to the demand for the HP15Ø for other experimental work, further analysis of the resulting data was carried out on a Sperry personal computer coupled to a six colour plotter.

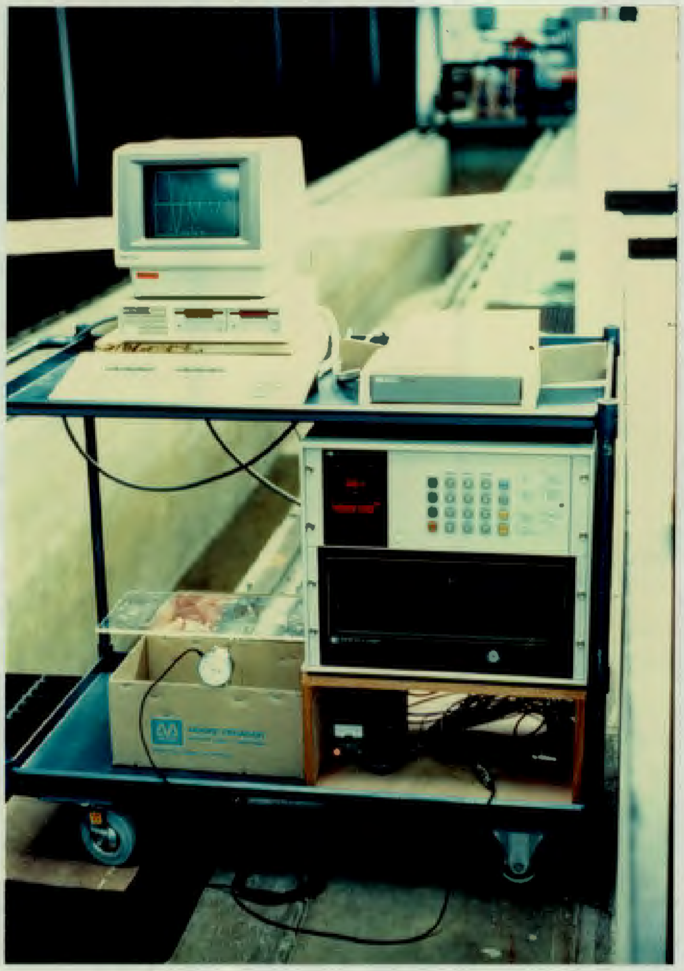


PLATE 4.1

ELECTRONIC EQUIPMENT

4.3 THE MODEL

4.3.1 CHOICE OF MODEL DIMENSIONS

A major objective of this thesis was to compare results obtained with those of:-

a) Blumberg 1983

b) Theoretical results from literature

c) Experimental results from literature

It was therefore necessary to use the same type of ship as Blumberg and others had used. (In fact Blumberg's original model was used, see 4.3.3).

In the early sixties, the Netherlands Ship Motions Basin in Wageningen published comprehensive test results on Series 60 hull forms, which therefore became the standard for further ship motion studies. The most common block coefficient at the time amongst supertankers was 0,7 and this appears to have also become a standard, although modern tankers now have block coefficients of 0,8 and over. Table 4.3.1 indicates some of the authors who have used series 60 hulls with a block coefficient of 0,7 for recent analytical or empirical investigations.

AUTHOR / SHIP PROPERTY	KIM (1968)	GERRITSMA and BEUKELMAN (1966)	BECK and TUCK (1971)	BEUKELMAN and GERRITSMA (1982)	BLUMBERG (1983)	BLAKE (1986)
L_{pp} (m)	3,Ø	2,258	213	2,256	2,Ø	2,Ø
B (mm)	428	322	3Ø,5x1Ø^3	322	285,7	285,7
T (mm)	171	129	12,2x1Ø^3	129	114,3	114,3
V (m^3)	Ø,154	Ø,Ø67	54,5x1Ø^3	Ø,Ø657	Ø,Ø457	Ø,Ø457
K (m)	Ø,75	Ø,565	--	Ø,75	Ø,5	Ø,5

where K = the pitch radius of gyration

TABLE 4.3.1

AUTHORS WHO HAVE USED SERIES 6Ø HULL FORMS FOR ANALYTICAL OR EMPIRICAL INVESTIGATIONS

An analysis of the World Tanker Register shows that since 1965, a 100 000 dwt displacement is common among large tankers. It would therefore be logical to model tankers of not less than 100 000 dwt. It can also be seen from table 4.3.2 that above 100 000 dwt, the dimensional ratios of tankers are reasonably constant.

DISPLACEMENT x1000 dwt	L_{pp}/T	B/T
70 to 99	18,31	2,91
100 to 199	16,50	2,63
200 to 239	16,00	2,44
> 240	16,10	2,65

TABLE 4.3.2

RATIOS OF PRINCIPAL SHIP DIMENSIONS

where :-

L_{pp} = Length between perpendiculars

B = Extreme breadth

T = Extreme draught

4.3.2 SCALE OF THE MODEL

The scale used for the model ship was governed by the width of the towing channel; ie 1,52 m. During the original vertical motion tests by Blumberg, it was essential that the ship did not interact with the reflection of its own bow wave when moving at the lowest Froude number. A scale of 1:144 was finally selected, the fact that it is a perfect square simplifying prototype-model conversions.

For ideal modelling of prototype motions, it would be necessary to maintain both Froude and Reynold s' numbers the same in the model as they are in the prototype. Froude's number is given by :

$$Fr = \frac{\text{SHIP VELOCITY}}{\sqrt{\text{SHIP LENGTH} * \text{GRAVITY}}}$$

and Reynolds' number by :

$$Re = \frac{\text{SHIP VELOCITY} * \text{SHIP LENGTH}}{\text{KINEMATIC VISCOSITY}}$$

It is not possible to scale both the gravity factor from Froude and the viscosity factor from Reynolds simultaneously. Hence it is standard practice to model the prototype according to Froude's law and then, if necessary, compensate for dissimilar viscosity effects. Tests have been carried out by the National Research Council, Ottawa, Canada, where a long chain polymer was added to the water to reduce the viscosity effects. This proved to be impractical however, as the change in viscosity was small and the polymer would adhere to the ship hull, necessitating cleaning after every test.

Using the 1:144 scale and the dimensional ratios obtained from table 4.3.2, the characteristic dimensions of the model are :-

Length between perpendiculars	L_{pp}	=	2,Ø m
Breadth amidships	B	=	285,7 mm
Draught amidships	T	=	114,3 mm
Displacement	V	=	45,718 dm^3
Block coefficient	C_b	=	Ø,7

$$L_{pp}:B:T \quad = \quad 17{,}5Ø:2{,}5Ø:1$$

Sectional properties and the lines plan of the hull may be found in Appendix A.

4.3.3 CONSTRUCTION OF THE MODEL

4.3.3.1 ORIGINAL CONSTRUCTION OF THE MODEL

The model was made of Jelutong, a fairly soft wood, which facilitated shaping of the bow and stern. It was constructed out of seven laminations, glued together with a waterproof wood glue. A hold area was milled out of the deck, and was later deepened during drag tests in 1984. In the hold parallel rails were fitted, on which steel ballast weights could slide. The stern drive location was milled out such that any thrust transmitted via the rear suspension arm would approximate the position of the propeller shaft in the prototype.

A perspex splash cover was fitted over the hold area and a splash shield around the stern drive hole. The model was treated with an oil based water proof primer and finally painted with a clear sealant. Leakages during initial tests were resealed using fibre glass.

4.3.3.2 MODIFICATIONS TO THE MODEL

To emphasise the water line, the freeboard was painted signal red and the hull white. It was found during tests that it would have been easier to see the water line if the colours had been reversed, having the darker colour below water.

Although the original sliding masses fitted by Blumberg were still available, additional ballast had to be added to bring the ship back up to its design mass of 45,718 kg. This was due to the fact that the hold area had been milled to almost twice the original depth to take instrumentation for the drag tests.

For the pure pitch motion tests, the model had to be pivoted about its centre of gravity. To facilitate this, two teflon bearings were made and set into the hull of the ship, corresponding with the centre of gravity and the water line. (See plate 4.2).

During the final set of tests, the model started to delaminate due to leakage. This problem was solved temporarily by 'locking' the laminations together with 120 mm long wood screws, from both top and bottom, aft and fore of the model. Every time repair work was carried out on the model, the centre of gravity had to be checked and if necessary reset. If the centre of gravity was found to need significant correction, then the moment of inertia of pitch in air also had to be checked and if necessary corrected.

4.3.4 GEOMETRICAL PROPERTIES OF THE MODEL

The relative positions of the centre of buoyancy of the model and prototype had to be kept the same if motions were to be modelled with any accuracy. According to the lines plan, the longitudinal centre of buoyancy would lie at 2,5Ø6% of the ship length forward of amidships, assuming homogeneous construction. Thus, if the model was to float on an even keel, the centre of gravity had to lie on a vertical line through the centre of buoyancy. This was achieved by moving ballast appropriately, such that the model balanced on a transverse fulcrum placed below the centre of buoyancy. As roll motions were not to be studied, the vertical position of the centre of gravity was not critical and for convenience was assumed to lie on the water line.

Besides the centre of gravity, the pitch radius of gyration also had to be set to the equivalent of the prototype, ie 25% of length between perpendiculars. This could be done theoretically by integrating over the length of the model, but to obtain a more accurate result it was decided to rather use an indirect empirical approach.

Consider a model suspended by two springs of known stiffness, equidistant about the centre of gravity and pinned at the centre of gravity such that it can only move in a pitching motion about a transverse axis through the centre of gravity. (See figure 4.3.4).

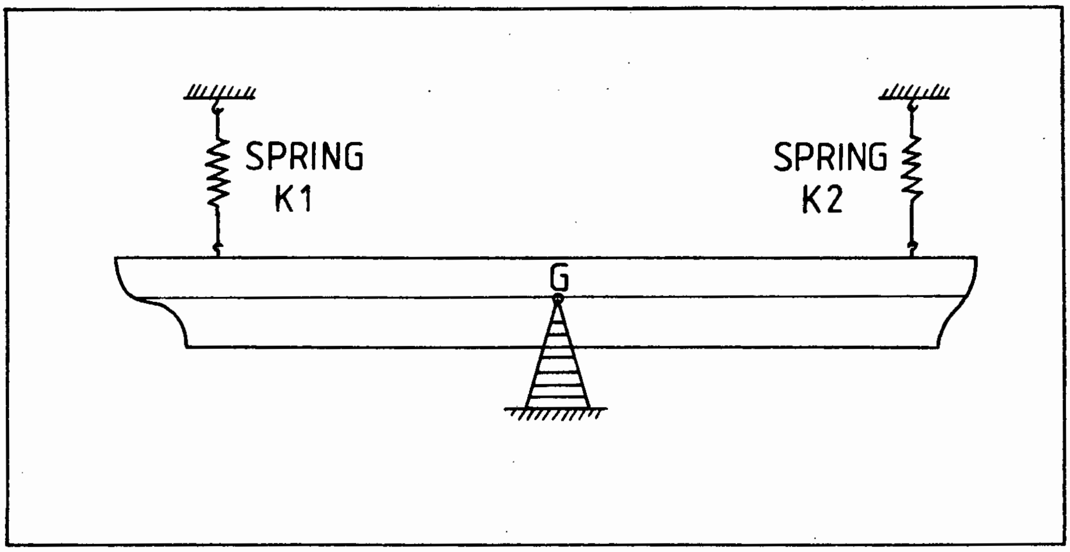


FIGURE 4.3.4

CONFIGURATION FOR PITCH MOMENT

OF INERTIA IN AIR

PLATE 4.2

TEFLON BEARING AND PIVOT AT CENTRE OF GRAVITY FOR PITCH

PLATE 4.3

SHIP SUSPENDED IN FRAMEWORK TO FIND THE PITCH MOMENT OF INERTIA IN AIR

On displacing the stern or bow and then releasing the model, it will oscillate in a pitch motion which can be described by the equation :-

$$\ddot{\alpha}.I = R$$

where

α = angle of pitch

I = pitch moment of inertia = Mr^2

R = restoring moment

r = pitch radius of gyration

M = mass of model

If the springs have stiffnesses K1 and K2 and assuming small angles of rotation α, then :-

$$R = -a^2.\alpha.(\ K1 + K2\ )$$

where 'a' is the axial distance from the centre of gravity to the springs

Hence the undamped pitch period in air T_p' is given by:-

$$T_p' = 2.\pi.\sqrt{\frac{I}{(\ K1 + K2\ ).a^2}}$$

Two coil springs were calibrated, (see appendix D) and found to have spring constants of K1 = 1713,8 N/m and

K2 = 1776,2 N/m. The pitch moment of inertia of this model was found to be

$$I = 11,430 \text{ kg.m}^2$$

giving an undamped pitch period of

$$T_p' = 0,556 \text{ s.}$$

This period was set by moving the inboard sliding weights appropriately, taking care at the same time to maintain symmetry and hence the correct position of the centre of gravity.

4.3.5 SUPPORT FRAMEWORKS FOR THE MODEL

4.3.5.1 FRAMEWORK FOR PURE MOTION TESTS

A scaffolding frame was set up in the centre of the wave basin across which a channel beam was clamped horizontly. From this beam, three vertical stanchions could be bolted, corresponding with the centre of gravity of the model and the two support points, 645 mm either side of the centre of gravity. (See plate 4.4). These stanchions gave a clearance of 250 mm above the water surface when the water depth was at its maximum of 285,8 mm. The LDT's were clamped to these stanchions

PLATE 4.4

VIEW OF THE FRAMEWORK
AROUND THE MODEL

PLATE 4.5

SHIP SUSPENDED IN FRAMEWORK FOR
PURE PITCH MOTIONS

and in the case of heave motions, linear bearings were bolted to the centre stanchion. To supply the necessary pivot points for pitch motions, an auxillary frame was placed around the model and held in position by weighing it down with concrete cubes. This system was used for all water as well as the air tests. (See plate 4.5).

4.3.5.2 FRAMEWORK FOR COUPLED MOTION TESTS

To measure coupling coefficients, it was necessary for the model to be able to move freely in both heave and pitch motions. This was achieved by employing a system of pantographs, originally used by Blumberg. Lateral support was afforded by the bow parallelogram only.

The pantograph was made from 20 mm square aluminium section of wall thickness 2 mm. All bearing housings were of milled or cast aluminium, to take 19 mm bearings. The cantilever in the stern pantograph was later replaced with an 18 mm square steel section to improve the stiffness of the system. Both components were connected to the vertical supports by slides, so that they could be raised or lowered with the variable water depths during tests.

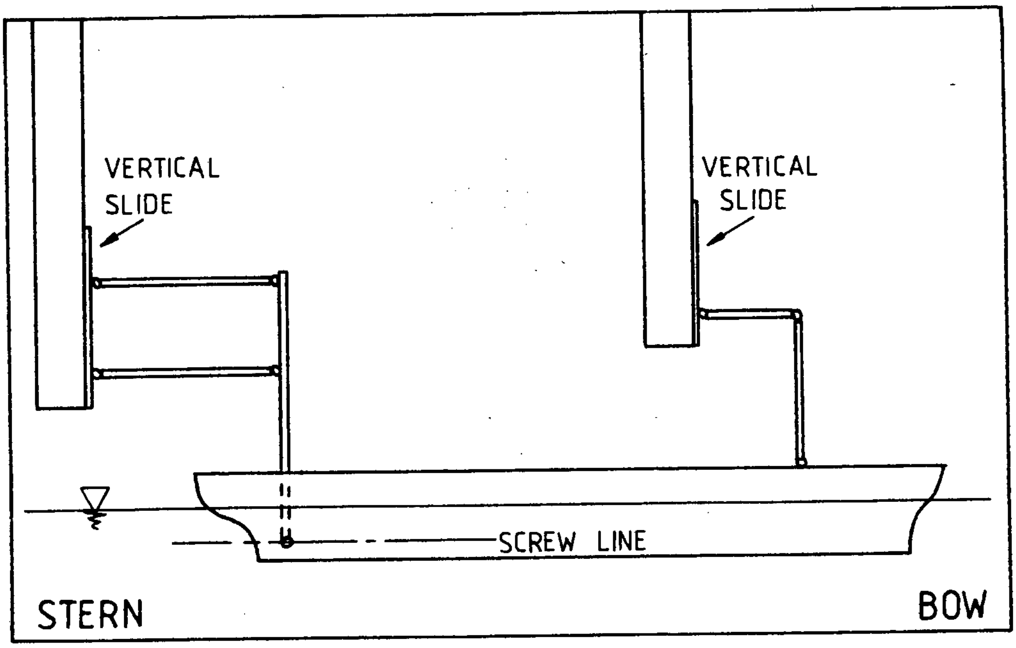


FIGURE 4.3.5

SKETCH OF PANTOGRAPH SUPPORT SYSTEM

4.4 PITCH MOTIONS

Pitch motion tests were started at the shallowest of the five test depths, ie D/T = 1,1 or D = 125,7 mm. However at this depth, the LDT's were not long enough to reach the ship deck from the support stanchions. Thus it was necessary to manufacture two 100 mm long extensions which could be screwed on to the ends of the LDT's.

The ship was restricted to pure pitch motions by pivoting it at the centre of gravity using the auxillary frame as described in 4.3.5.1.

The model was given an initial displacement by lowering a 3,1 kg mass on to the aft deck via a fishing trace wire which could be controlled from the edge of the basin. (As can be seen in plate 4.5). Once the water in the basin had settled down again, the mass was quickly raised, effectively setting the model into a pitch oscillation which could be recorded. This method was used in order that a constant initial displacement could be used throughout the tests. However in the case of D/T = 1,1 , the displacement induced was greater than the clearance under the keel and the ship grounded. As the water damping was great enough to prevent further grounding once in motion, this was not considered a problem. Ten sets of readings were taken at each water depth and the average results found.

The intial tests gave results that were so damped that any attempt to take data from them would be erroneous. Thus tension springs of known spring constant were added to the system, in an attempt to increase the water spring effect. (See plate 4.6). The results of these tests were significantly better and the springs were used throughout the pitch tests.

It was also found that the foam rubber wave absorbers fitted to the basin, were not as effective as had been hoped. Hence it took 3-4 minutes for the water to settle after each test.

PLATE 4.6

VIEW OF SPRING SYSTEM USED FOR PURE PITCH MOTION TESTS

4.5 HEAVE MOTIONS

To maintain pure heave motions, a stainless steel rod was bolted perpendicularly to a flat plate which was fixed to the deck of the model, centered over the centre of gravity, such that the rod pointed vertically upwards. Two linear bearings were bolted to the machined surface of the centre stanchion, through which the rod could slide. (See plate 4.7). Thus the model could effectively move only vertically, any pitching moment being taken by the rigidity of the steel plate holding the vertical rod. The mass of the stainless steel rod and mounting plate were subtracted from the ballast, to maintain the ship at its design mass.

This system gave no restrictions against yaw, but as there is no coupling between yaw and heave motions, there was no induced yaw motion during tests. Some did occur after each test however, due to unsymmetric reflected waves and to prevent this two strings were attached to the aft bollard, anchored to the framework.

As with the pitch tests, an intial displacement was made by lowering a 3,1 kg mass onto the deck. The mass was positioned as close as possible to the centre of gravity so as to minimise any unnecessary pitch moments. Ten sets of readings were taken for each water depth and the average results found.

It was found that adequate results were obtained without the use of any additional springs in the system.

4.6 COUPLED MOTIONS

For the coupled motion tests the model had to be free to move in both pitch and heave motions. Thus the pantograph system described in 4.3.5.2 was used to restrict unwanted motions. (See plate 4.8). Due to the relationship that heave and pitch coupling coefficients are equal for a ship with no forward velocity, it is theoretically possible to obtain the required data from either of two methods;

a) Displacing the model in pitch and monitoring the resultant heave motion.

b) Displacing the model in heave and monitoring the resultant pitch motion.

However, the practicality of displacing a model in pitch without applying heave displacement simultaneously is difficult and hence the second method was used.

Unlike the pure heave motion tests, there was nothing to prevent pitch on intial displacement of the model, so the positioning of the mass was crucial. It was found that if the mass was placed 2mm behind the centre of gravity, the model floated on an even keel in the displaced position. The LDT's were set up to monitor the motion as for the pure pitch tests, except that in this case the motion measured was a combination of pitch and heave. To extract the relevant pitch motion, the difference was found between the two displacements, (fore and aft), to give the difference in elevation between the two support points on the deck, or twice the pitch.

PLATE 4.7

VIEW OF BEARING SYSTEM RESTRICTING

MOVEMENT TO HEAVE ONLY

PLATE 4.8

VIEW OF PANTOGRAPH SUPPORT FRAMEWORK

FOR COUPLED HEAVE AND PITCH

4.7 WAVE EXCITING FORCE TESTS

To measure the exciting foces on a ship due to oncoming waves, the model had to be held rigidly whilst subjected to waves. To this end, the model was supported using the pantograph system, and then fixed in position by vertical rods from the support points, to cantilevers attached to the framework, the framework being supported by the towing trolley over the wave channel. (See plates 4.9 and 4.1Ø).

Any strain in the cantilevers would give a measure of the force imparted to them from the model via the vertical rods. As the exciting force is a function of the wave height, this variable also had to be accurately measured. This was done using a standard twin wire probe made of Ø,2 mm diameter stainless steel wire. The probe was connected to an electronic unit which gave an output voltage directly proportional to the depth of immersion of the probe. The probe was placed in the centre of the channel, four metres ahead of the model, so as not to pick up any effects from the model itself.

Although the wave generator was capable of generating a wide range of wave heights, to keep the waveform sinusoidal and avoid any reflection of waves, the wave height was restricted to being less than 30 mm.

To set the period of the wave, a counter attached to the wave paddle was used. The time for 100 oscillations would be taken and the period calculated. Although specific periods were desired, it was not possible to set the wave generator to give the exact period and hence values within 0,05 s of that desired were accepted. As with the pure motion tests, each test was repeated ten times and average results found.

4.8 WATER DEPTHS AND WAVE PERIODS

Five water depths were used for each type of test as shown in table 4.8A. These depths were chosen to correspond to those used by other authors, in particular Blumberg (1983), so that comparisons could be drawn later.

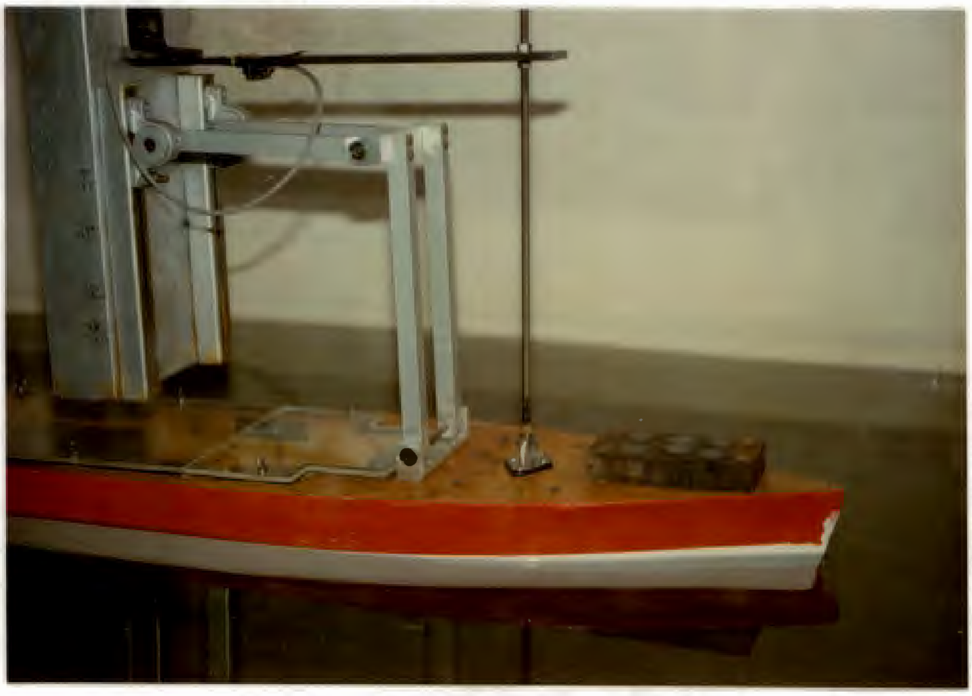


PLATE 4.9

VIEW OF FORE PANTOGRAPH SYSTEM

AND SUPPORT CANTILEVER

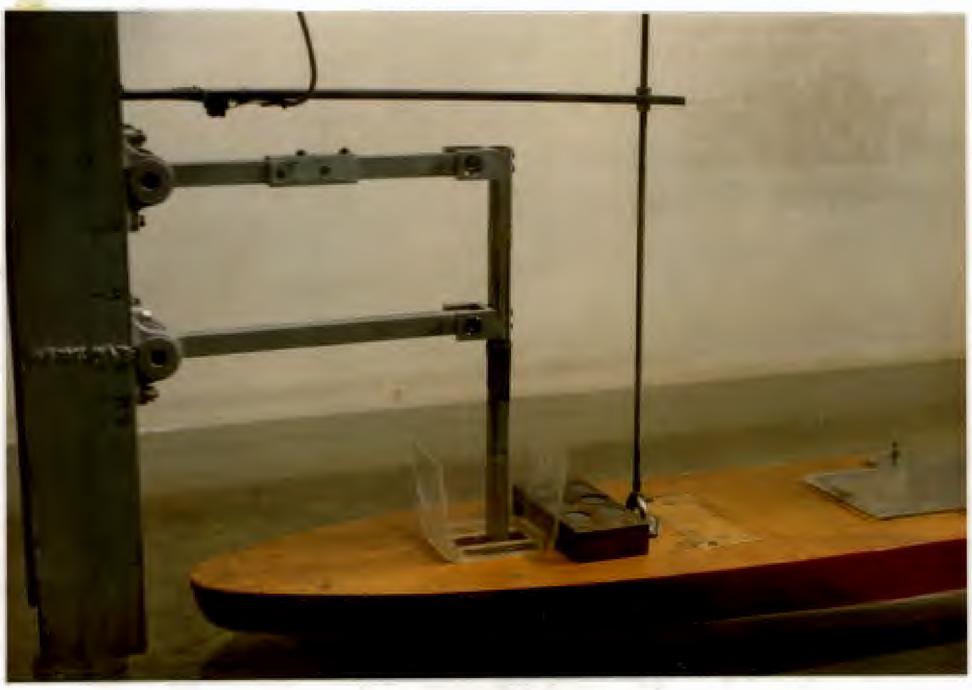


PLATE 4.10

VIEW OF AFT PANTOGRAPH SYSTEM

AND SUPPORT CANTILEVER

	DEPTH TO DRAUGHT RATIO				
	1,1	1,2	1,5	2,0	2,5
MODEL WATER DEPTH (mm)	125,7	137,2	171,5	228,6	285,8
PROTOTYPE WATER DEPTH (m)	18,11	19,75	24,69	39,92	41,15

TABLE 4.8A

WATER DEPTHS USED IN THIS THESIS

The wave periods chosen for the wave exciting force tests represent a compromise between wave periods encountered at sea and the wave generator capability. The periods used, (or as close to these values as possible), are given in table 4.8B.

MODEL PERIOD (s)	1.00	1.20	1.40	1.60	1.90
PROTOTYPE PERIOD (s)	12,0	14,4	16,8	19,2	22,8

TABLE 4.8B

WAVE PERIODS USED IN THIS THESIS

5 ANALYSIS OF EXPERIMENTAL DATA

The pitch, heave and coupled motion tests in still water produced graphical results in the form of displacement versus time. From these graphs, the damped period of oscillation and the damping factor had to be found. With this information, it was then possible to calculate the viscous damping coefficient and virtual mass or moment of inertia.

5.1 METHOD OF ANALYSIS OF MOTION TESTS

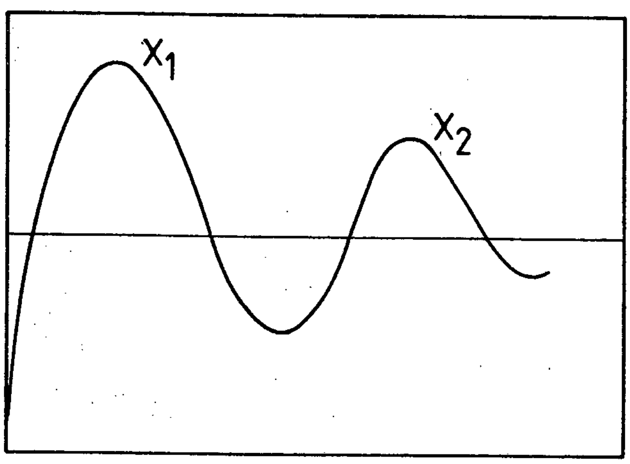


FIGURE 5.1

TYPICAL MOTION OSCILLATION

From vibrational analysis it can be shown that for a damped sinusoidal motion, as shown in figure 5.1, the amplitude

$$X_{n+1} = X_n . \exp\left[2.\pi . \frac{(-b)}{\sqrt{1-b^2}}\right]$$

where "b" is the ratio of the damping to the critical damping. The critical damping can be defined as the damping at the transition between the case where a body oscillates about the equilibrium position and the case where the body simply moves back to its equilibrium position without oscillating. In the case of the ship motions under study, the damping is less than critical and therefore the damping factor "b" is always less than unity.

Simplifying the equation:-

$$\ln \frac{X_{n+1}}{X_n} = 2.\pi . \frac{(-b)}{\sqrt{1-b^2}}$$

$$\text{letting } Q = \frac{1}{2.\pi} . \ln \frac{X_{n+1}}{X_n}$$

$$\frac{-b}{\sqrt{1-b^2}} = Q$$

$$b = \sqrt{\frac{Q^2}{(1 + Q^2)}}$$

By definition

$$b = \frac{c}{C_c}$$

and the critical damping

$$C_c = 2.\sqrt{K.M}$$

therefore the damping is given as

$$c = 2.b.\sqrt{K.M}$$

For heave motions the damping is then

$$B_{33} = 2.b.\ \sqrt{\Sigma(K).A_{33}}$$

or for pitch motions

$$B_{55} = 2.b.\ \sqrt{\Sigma(K.a^2).A_{55}}$$

The damping coefficient for coupled motions is an intermediate function between these two, given as

$$B_{35} = 2.b.\ \sqrt{\Sigma(K.a).A_{35}}$$

The values of the Ajk terms can be found by rearranging the formula for the damped period of the system. Thus the virtual mass in heave is

$$A_{33} = \frac{T_h^2.\Sigma(K).(1-b^2)}{4.\pi^2}$$

and similarly the virtual moment of inertia in pitch is

$$A_{55} = \frac{T_p^2.\Sigma(K.a^2).(1-b^2)}{4.\pi^2}$$

For the case of coupled motions, this value is an intermediate value between virtual mass and virtual moment of inertia given as

$$A_{35} = \frac{T_c^2.\Sigma(K.a).(1-b^2)}{4.\pi^2}$$

The spring constants of all extra springs used, including the transducers (K3, K4, K5) were found. (See appendix D). As well as these springs the action of the water restoring force as a "spring" in the system was also found experimentally for both heave and pitch motions. The values of these constants are

Heave ; $K_{water} = 4444,4\ N.m^{-1}$

Pitch ; $K_{water} = 986,9\ N.m.rad^{-1}$

5.2 PITCH MOTION RESULTS

Figure 5.2A below shows the simplified spring configuration used in the pitch motion tests.

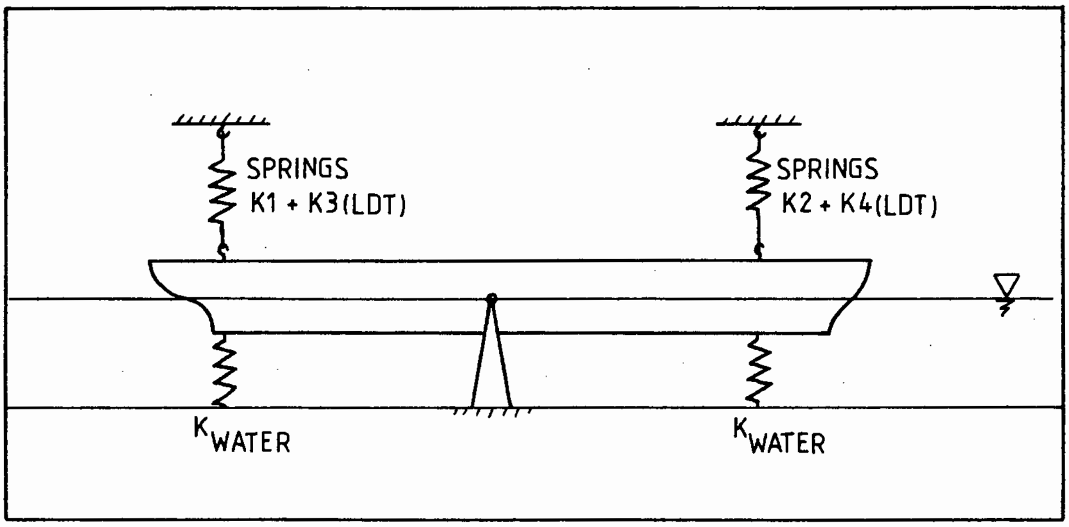


FIGURE 5.2A

SPRING CONFIGURATION IN PITCH MOTION TESTS

Using the formulae given in the previous section, the added moments of inertia and the damping coefficients for the five different water depths were calculated and are given in table 5.2 and plotted graphically in figure 5.2B. The extra case of the ship pitching in air is used as a check on the moment of inertia of the ship in pitch. This was found to be within 3,5% of that of the prototype which, allowing for experimental error, is acceptable.

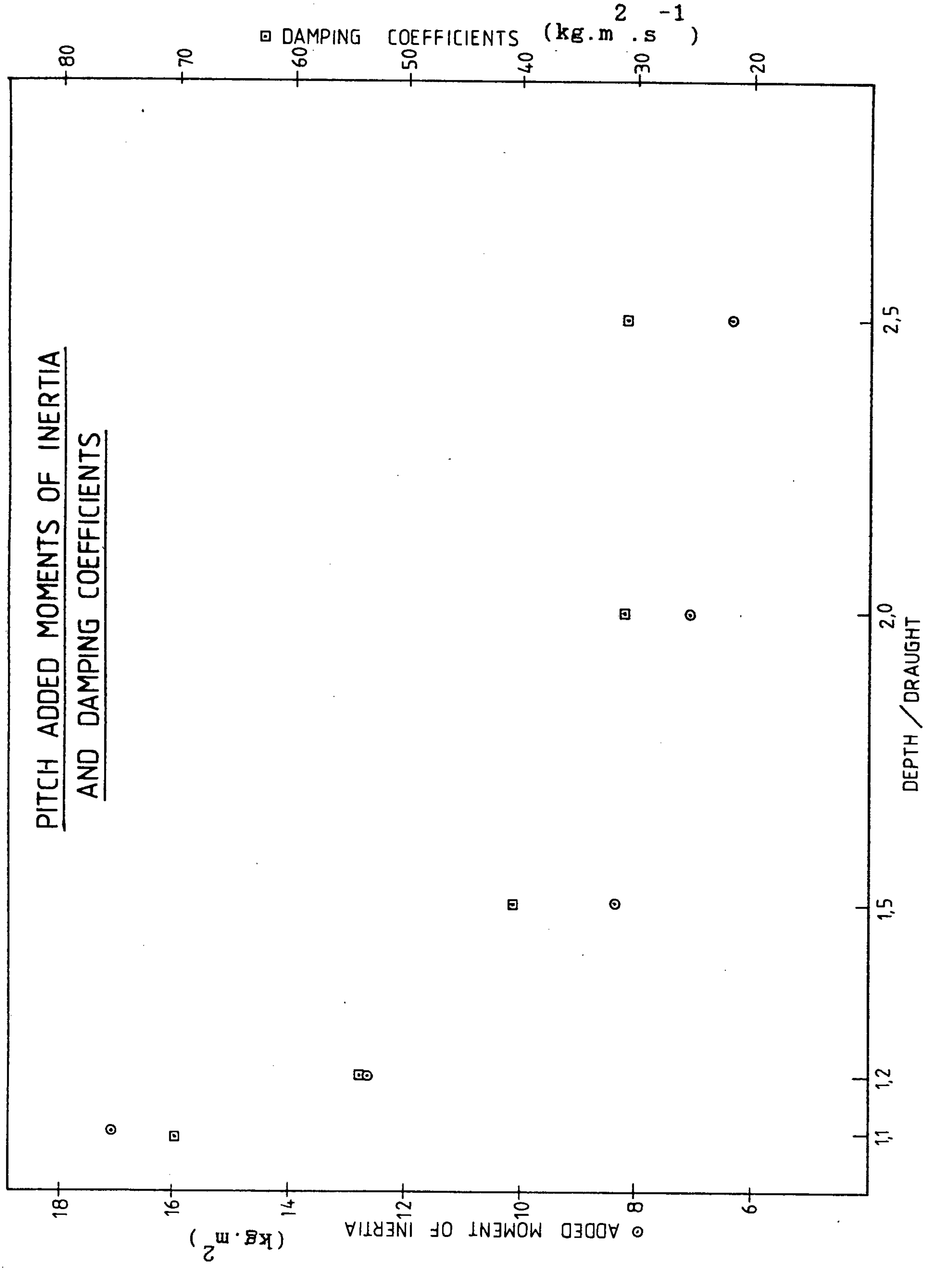
$^{d}/_{T}$	T_p (s)	AMP. RATIO	DAMP. FACT.	DAMP. COEFF. $kg.m^2/s$	VIRT. INERTIA $kg.m^2$	ADDED INERTIA $kg.m^2$
1,1	Ø,681	Ø,43	Ø,134	71,28	28,52	17,Ø9
1,2	Ø,624	Ø,48	Ø,116	56,59	24,Ø6	12,63
1,5	Ø,566	Ø,54	Ø,Ø98	43,27	19,87	8,44
2,Ø	Ø,546	Ø,62	Ø,Ø76	32,5Ø	18,56	7,13
2,5	Ø,536	Ø,62	Ø,Ø77	32,33	17,89	6,46
AIR	Ø,561	Ø,89	Ø,Ø19	4,97	11,84	---

TABLE 5.2

PITCH ADDED MOMENTS OF INERTIA

AND DAMPING COEFFICIENTS

FIGURE 5.2B


PITCH ADDED MOMENTS OF INERTIA
AND DAMPING COEFFICIENTS
DAMPING COEFFICIENTS (kg.m2.s-1)
80
70
60
50
40
30
20
ADDED MOMENT OF INERTIA (kg.m2)
18
16
14
12
10
8
6
DEPTH / DRAUGHT
1,1
1,2
1,5
2,0
2,5

5.3 HEAVE MOTION RESULTS

Figure 5.3A below shows the simplified spring configuration used in the heave motion tests.

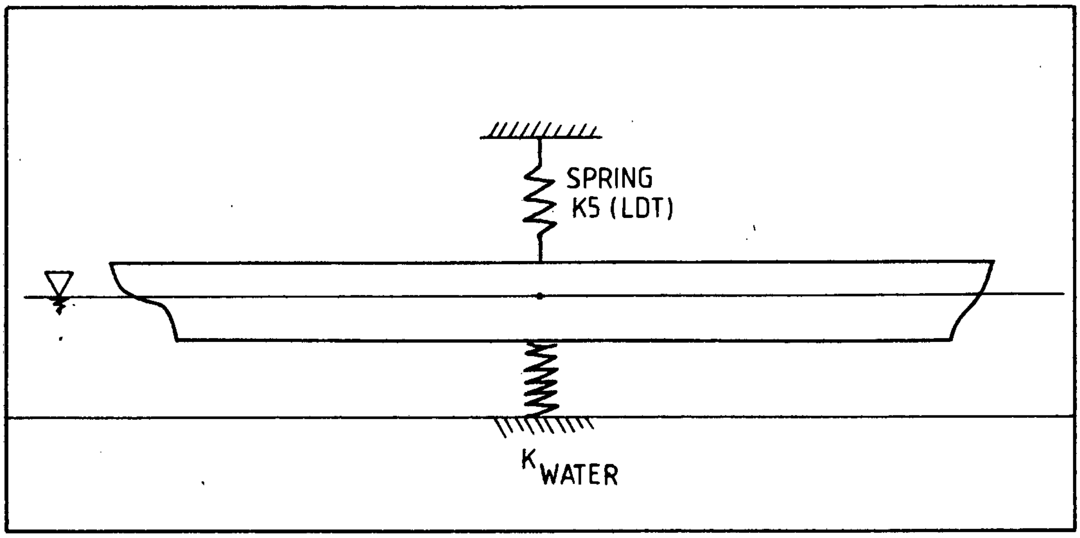


FIGURE 5.3A

SPRING CONFIGURATION IN HEAVE MOTION TESTS

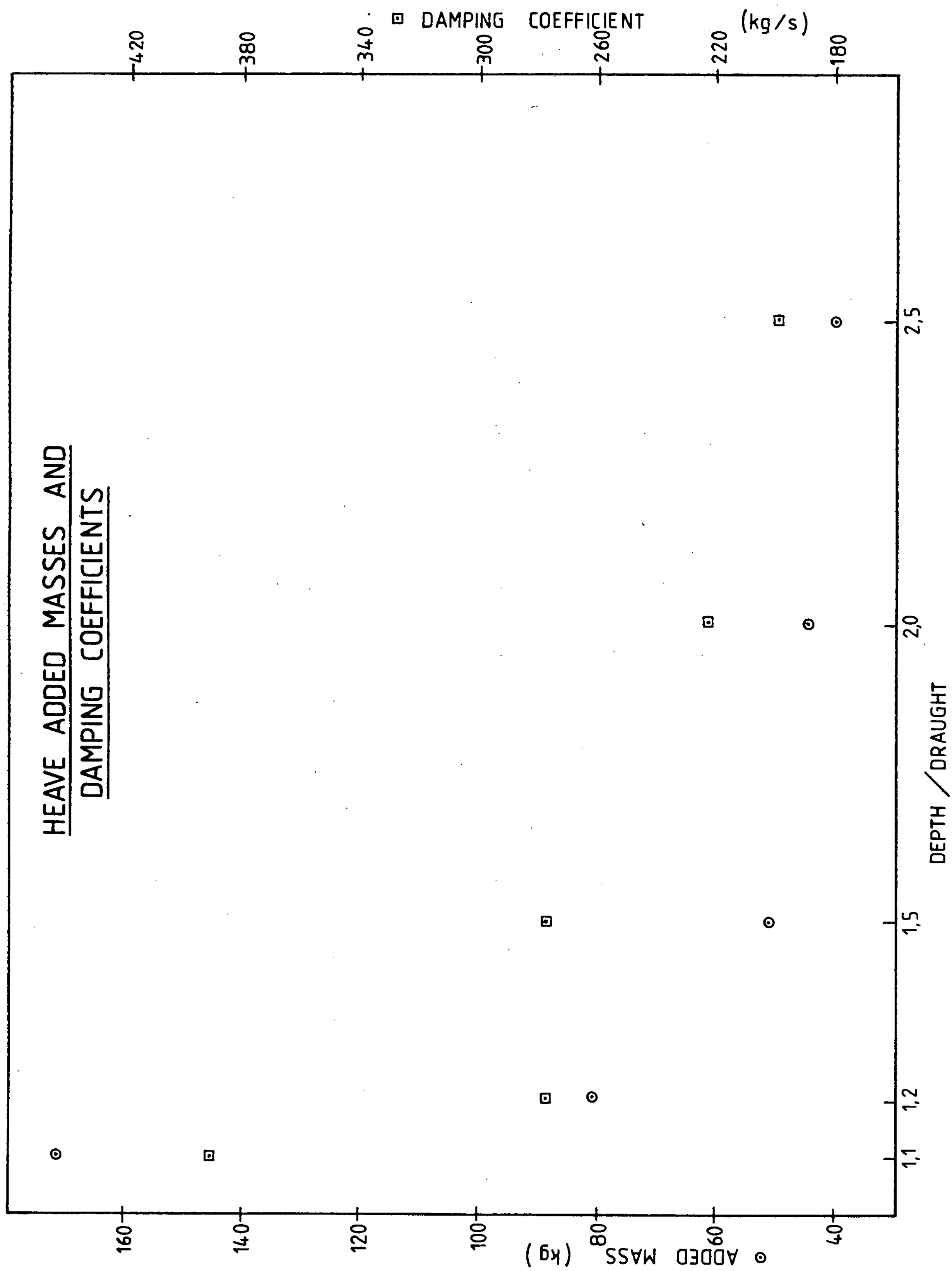
Using the formulae given in section 5.1 the added masses and the damping coefficients for the five different water depths were calculated and are given in table 5.3 and plotted graphically in figure 5.3B. The extra case of the ship heaving in air is used as a check on the mass of the ship. This was found to be within 1% of the actual mass of 45,718 kg.

$^{d}/_{T}$	T_h (s)	AMP. RATIO	DAMP. FACT.	DAMP. COEFF. (kg/s)	VIRT. MASS (kg)	ADDED MASS (kg)
1,1	1,238	Ø,33	Ø,174	391,6	218,61	172,89
1,2	1,Ø74	Ø,31	Ø,183	277,5	127,23	81,52
1,5	Ø,942	Ø,26	Ø,21Ø	276,9	96,83	51,11
2,Ø	Ø,9Ø6	Ø,33	Ø,174	222,4	9Ø,86	45,14
2,5	Ø,882	Ø,36	Ø,16Ø	2ØØ,4	86,5Ø	4Ø,79
AIR	Ø,716	Ø,93	Ø,Ø12	9,6	46,11	---

TABLE 5.3

HEAVE ADDED MASSES AND DAMPING COEFFICIENTS

FIGURE 5.3B


HEAVE ADDED MASSES AND
DAMPING COEFFICIENTS
DAMPING COEFFICIENT (kg/s)
420
380
340
300
260
220
180
ADDED MASS (kg)
160
140
120
100
80
60
40
DEPTH / DRAUGHT
1,1
1,2
1,5
2,0
2,5

5.4 COUPLED MOTION RESULTS

The coupled motion tests measured pitch motions from an original heave displacement, thus the spring configuration is similar to that used for the pitch motion tests.

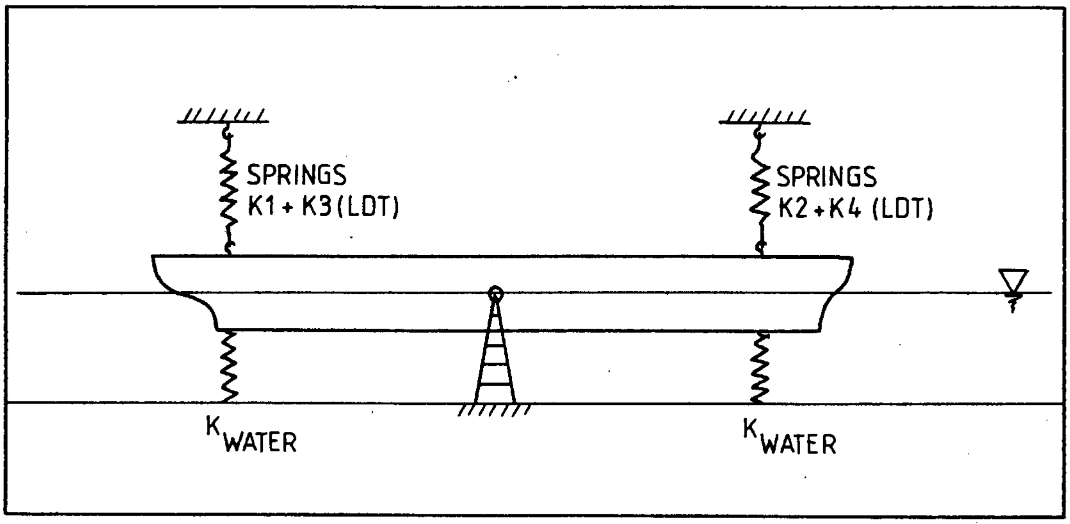


FIGURE 5.4

SPRING CONFIGURATION IN COUPLED MOTION TESTS

Using the formulae given in section 5.1 the various coefficients for the five different water depths were calculated and are given in table 5.4.

$^{d}/_{T}$	T_c (s)	AMP. RATIO	DAMP. FACT.	DAMP. COEFF. kg.m/s	VIRT. COUPLING kg.m	ADDED
1,1	1,37	Ø,16	Ø,28Ø	Ø,2ØØ	-Ø,Ø75	1,478
1,2	1,18	Ø,24	Ø,221	Ø,138	-Ø,Ø57	1,496
1,5	Ø,88	Ø,34	Ø,169	Ø,Ø8Ø	-Ø,Ø33	1,52Ø
2,Ø	Ø,81	Ø,51	Ø,1Ø7	Ø,Ø47	-Ø,Ø28	1,525
2,5	Ø,82	Ø,57	Ø,Ø89	Ø,Ø4Ø	-Ø,Ø29	1,524

<u>TABLE 5.4</u>

<u>COUPLED MOTION COEFFICIENTS</u>

5.5 WAVE EXCITING FORCE RESULTS

Unlike the motion tests, the wave exciting forces on the bow and stern of the ship could be read off the graphs directly. From these two forces it is possible to calculate the heave exciting force and the pitch exciting moment using the following relationships

Heave exciting force = $(F_{aft} + F_{fore})$

Pitch exciting moment = $(F_{aft} - F_{fore}).a$

where F_{fore} = force on the fore cantilever

F_{aft} = force on the aft cantilever

a = axial distance from centre of gravity to the point the force is measured at.

After the wave height and wave period were measured, the associated wavelength was calculated using the Airy theory as described in section 2.1.

Tests were run for each combination of the five water depths and the five wave periods, giving twenty-five tests in all. To differentiate between tests the following codes were used:-

PERIOD	DEPTH TO DRAUGHT RATIO				
(s)	1,1	1,2	1,5	2,0	2,5
1,90	1-1	2-1	3-1	4-1	5-1
1,60	1-2	2-2	3-2	4-2	5-2
1,40	1-3	2-3	3-3	4-3	5-3
1,20	1-4	2-4	3-4	4-4	5-4
1,00	1-5	2-5	3-5	4-5	5-5

TABLE 5.5A

TEST REFERENCE NUMBERS

The results of the tests are summarised in table 5.5B. In this form the exciting forces and moments are specific to a particular wave height. However, as the assumption of linearity is used throughout, the force amplitudes can be divided by their respective wave amplitudes to give general forces per unit wave height. This is the form that the forces must be in for use in the computer program. These results are summarised in table 5.5C.

CODE	PERIOD (s)	WAVE HEIGHT (mm)	WAVE LENGTH (m)	EXCIT. FORCE (N)	EXCIT. MOMENT (N.m)	PHASE SHIFT (rad)
1-1	1,88	8,2	2,Ø89	5,8	1,6	2,Ø2
1-2	1,58	9,5	1,755	3,8	Ø,1	3,39
1-3	1,41	12,1	1,568	2,4	Ø,7	4,64
1-4	1,21	9,7	1,344	1,1	3,1	3,85
1-5	1,Ø2	11,2	1,134	3,Ø	2,6	2,56
2-1	1,88	1Ø,7	2,183	5,4	1,8	1,76
2-2	1,6Ø	9,8	1,858	4,3	Ø,4	1,72
2-3	1,38	12,2	1,6Ø2	2,4	Ø,7	4,81
2-4	1,22	8,3	1,416	2,4	3,4	3,75
2-5	1,Ø1	14,4	1,174	1,3	2,3	2,4Ø
3-1	1,94	14,7	2,512	7,1	4,1	1,83
3-2	1,59	14,Ø	2,Ø64	8,3	1,5	1,73
3-3	1,38	18,Ø	1,793	5,9	Ø,3	4,64
3-4	1,18	29,7	1,536	2,2	2,4	4,22
3-5	Ø,97	21,3	1,261	3,7	4,9	2,ØØ
4-1	1,87	13,2	2,8Ø1	7,Ø	4,8	1,63
4-2	1,59	12,8	2,382	7,6	2,7	1,86
4-3	1,41	14,6	2,112	7,7	1,5	1,91
4-4	1,19	18,6	1,784	4,6	Ø,7	4,68
4-5	Ø,97	17,Ø	1,454	Ø,6	2,4	5,6Ø
5-1	1,89	12,4	3,166	7,1	5,6	1,82
5-2	1,63	14,5	2,73Ø	8,Ø	4,4	1,87
5-3	1,41	18,2	2,362	8,8	3,1	1,94
5-4	1,23	13,7	2,Ø52	5,Ø	Ø,5	2,3Ø
5-5	1,Ø2	18,Ø	1,7Ø9	Ø,6	1,Ø	Ø,Ø2

TABLE 5.5B

WAVE EXCITING FORCE RESULTS

CODE	PERIOD (s)	FORCE AMP. / WAVE AMP. (N/m)	MOMENT AMP. / WAVE AMP. (N.m/m)
1-1	1,9	1414	377
1-2	1,6	802	23
1-3	1,4	390	115
1-4	1,2	235	639
1-5	1,0	536	464
2-1	1,9	1004	336
2-2	1,6	871	72
2-3	1,4	393	116
2-4	1,2	578	807
2-5	1,0	181	324
3-1	1,9	966	558
3-2	1,6	1191	214
3-3	1,4	656	33
3-4	1,2	148	158
3-5	1,0	347	455
4-1	1,9	1063	720
4-2	1,6	1189	419
4-3	1,4	1052	206
4-4	1,2	498	79
4-5	1,0	71	282
5-1	1,9	1152	927
5-2	1,6	1103	601
5-3	1,4	964	341
5-4	1,2	727	67
5-5	1,0	67	111

TABLE 5.5C

WAVE EXCITING FORCES PER UNIT WAVE HEIGHT

6 ANALYSIS OF RESULTS

Having obtained all the coefficients for the equations of motion, (see section 2.3.4), these were entered into the computer program SMISW and the equations solved.

The intial program results of motion amplitudes for given wave periods and water depths bore little correlation to any results published in literature. Thus all the actual coefficients were checked against those calculated by computer programs from literature. The significance of each coefficient on the solution of the motion equations was also investigated.

It was found that only the exciting forces and moments varied greatly from published results and that these coefficients also dominated the solution of the equations.

The original objective of the wave exciting force tests was to find the forces and moments acting on the ship hull due to wave action. This was done, as explained in section 4.7, by holding the ship rigid in waved water and measuring the forces imparted to the supports. In hindsight it became clear that the forces measured in this manner are not the same as those acting on a ship hull which is free to move.

If one considers a ship acted upon by long period waves such that the wavelength is comparable with the ship length, then a wave crest will force the ship in a upward direction to try and maintain the balance between buoyancy and weight. When the following trough then begins to act on the hull, it may still be moving upwards due to its momentum, causing the restoring force to be increased. In the case of the fixed ship, the hull would still be in its zero position when the wave trough acts upon it. Thus the restoring force and hence the exciting forces on the free ship would be greater than those on the fixed ship. This is consistent with the discrepancies found between these empirical and calculated exciting forces for long period waves.

As the exciting forces and moments found empirically could not be used, it was decided to calculate the forces using the strip theory formulae given in section 3.2. The only parameters required for these formulae besides the ship's physical dimensions, are the added mass and damping coefficient per unit length along the ship. By using the added masses and damping coefficients found experimentally, the solutions to the equations of motion would still be based only on empirical data. However, for both the added mass and heave damping coefficient, only the total values for the ship were known and not the value per unit length as required. These values then had to be distributed along the length of the ship.

6.1 DISTRIBUTION OF ADDED MASS AND HEAVE DAMPING

It would be incorrect to assume that the added mass and damping coefficient varied linearly with the distance along the ship, as the ship's cross-sectional properties change rapidly towards the ends. Intuitively the added mass should be related to the mass of the displaced water, therefore it would be logical to assume that the added mass of a given ship section was proportional to the displaced volume of that section. Similarly the viscous damping coefficient should be related to the wetted surface area of the hull. Thus it is assumed that the damping coefficient for a particular ship section is proportional to the wetted surface of that section.

Taking a section of thickness "dx", the added mass would be proportional to the cross-sectional area below the water line and the heave damping coefficient proportional to the wetted perimeter of the section.

6.2 SOLUTIONS OF THE EQUATIONS OF MOTION

Using the distributions for added masses and heave damping coefficients given in the previous section in conjunction with the values found experimentally, the exciting forces and moments were calculated. The original empirical values of the exciting forces are compared with those calculated in the above fashion in figures 6.1 to 6.4.

The final solutions to the equations of motions were then found using the program SMISW and presented in the form of Response Amplitude Operator curves. The heave motion amplitudes are given in this form for the different water depths in figures 6.5, 6.7, 6.9, 6.11 and 6.13 and the pitch amplitudes in figures 6.6, 6.8, 6.10, 6.12 and 6.14. Superimposed onto these graphs are the results, either empirical or theoretical, of other authors for comparison. To emphasise the effect of water depth on the motion amplitudes, all five graphs are superimposed in figures 6.15 for heave and 6.16 for pitch.

FIGURE 6.1

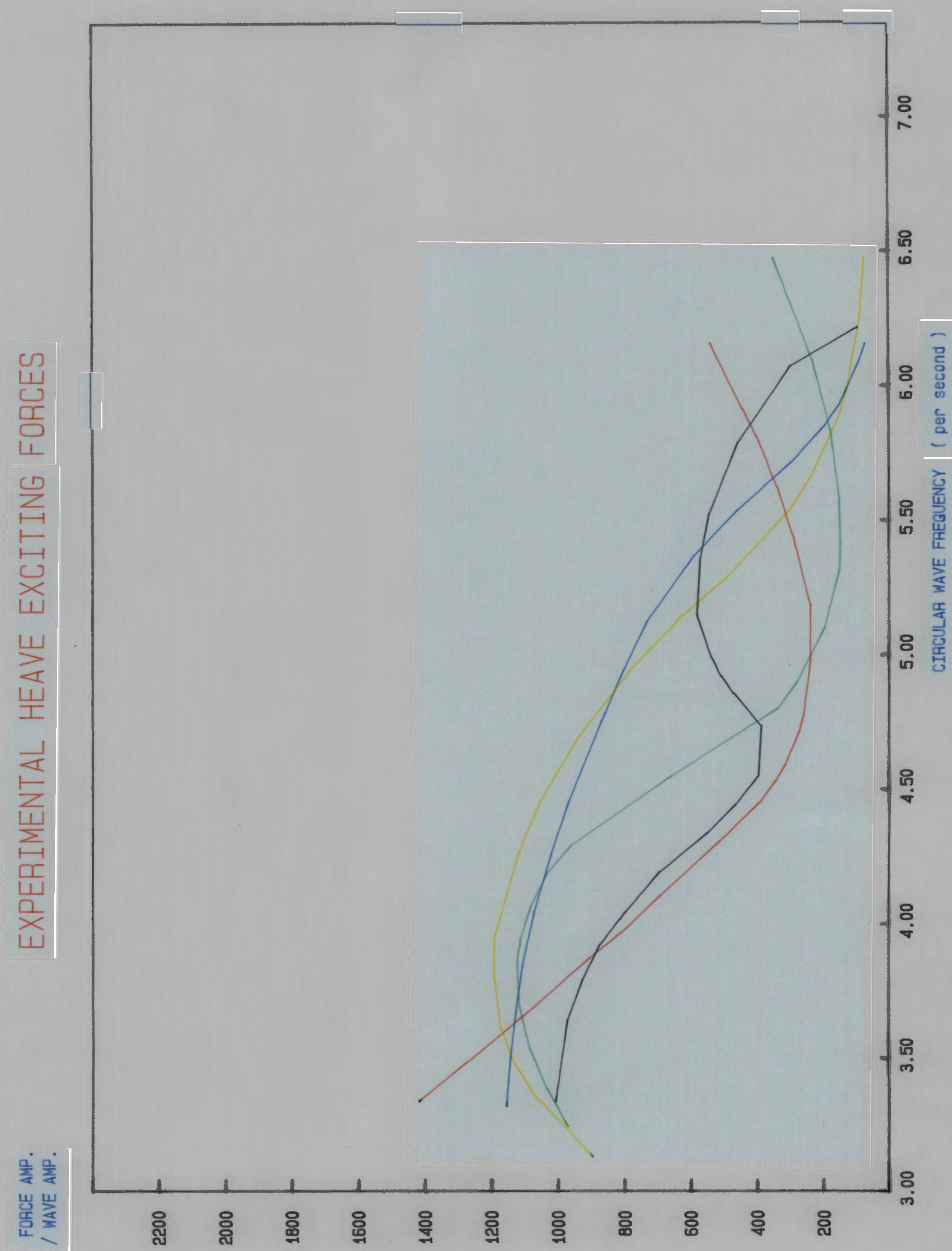

EXPERIMENTAL HEAVE EXCITING FORCES
FORCE AMP.
/ WAVE AMP.
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
3.00
3.50
4.00
4.50
5.00
5.50
6.00
6.50
7.00
CIRCULAR WAVE FREQUENCY (per second)

FIGURE 6.2

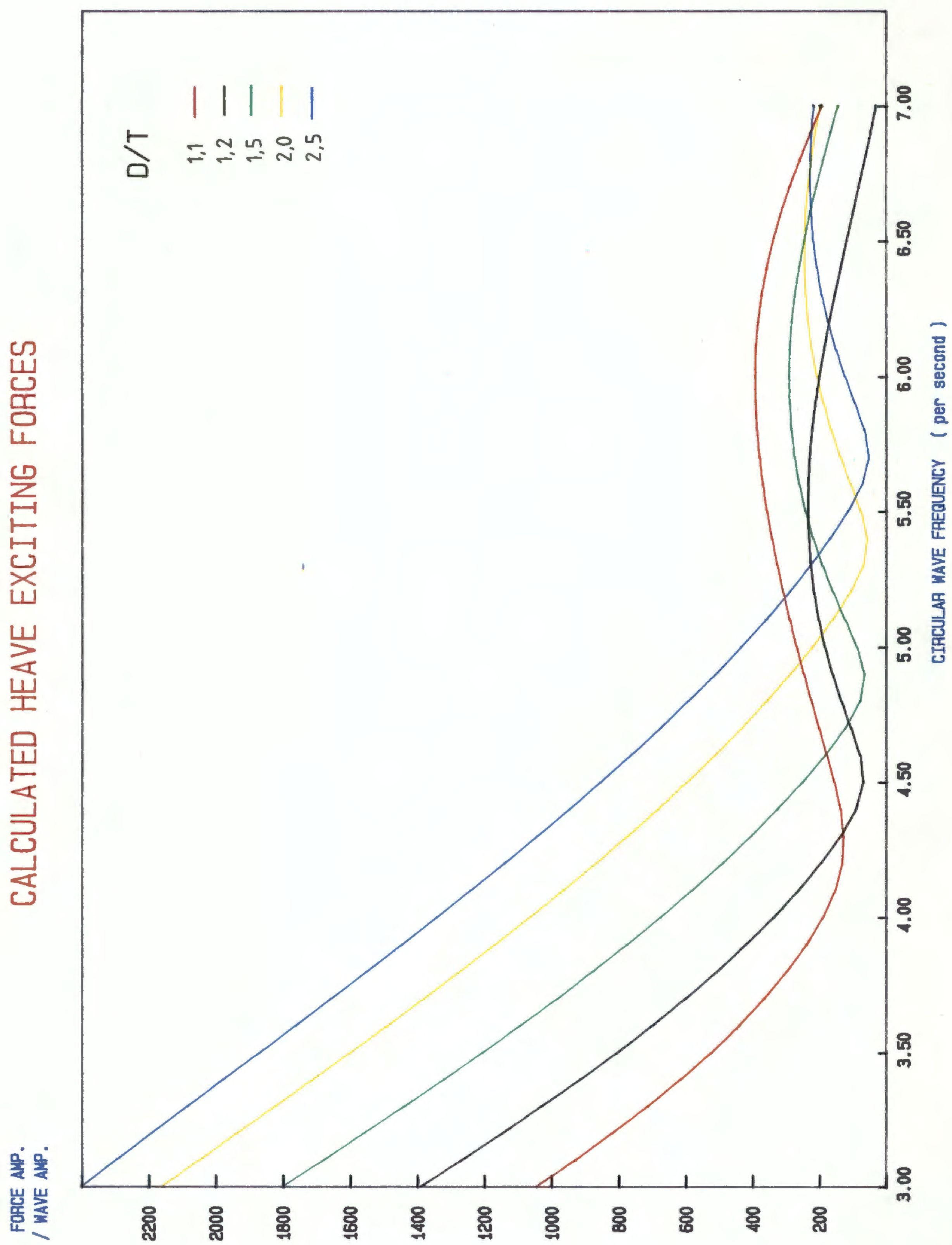

CALCULATED HEAVE EXCITING FORCES
FORCE AMP. / WAVE AMP.
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
3.00
3.50
4.00
4.50
5.00
5.50
6.00
6.50
7.00
CIRCULAR WAVE FREQUENCY (per second)
D/T
1,1
1,2
1,5
2,0
2,5

FIGURE 6.3

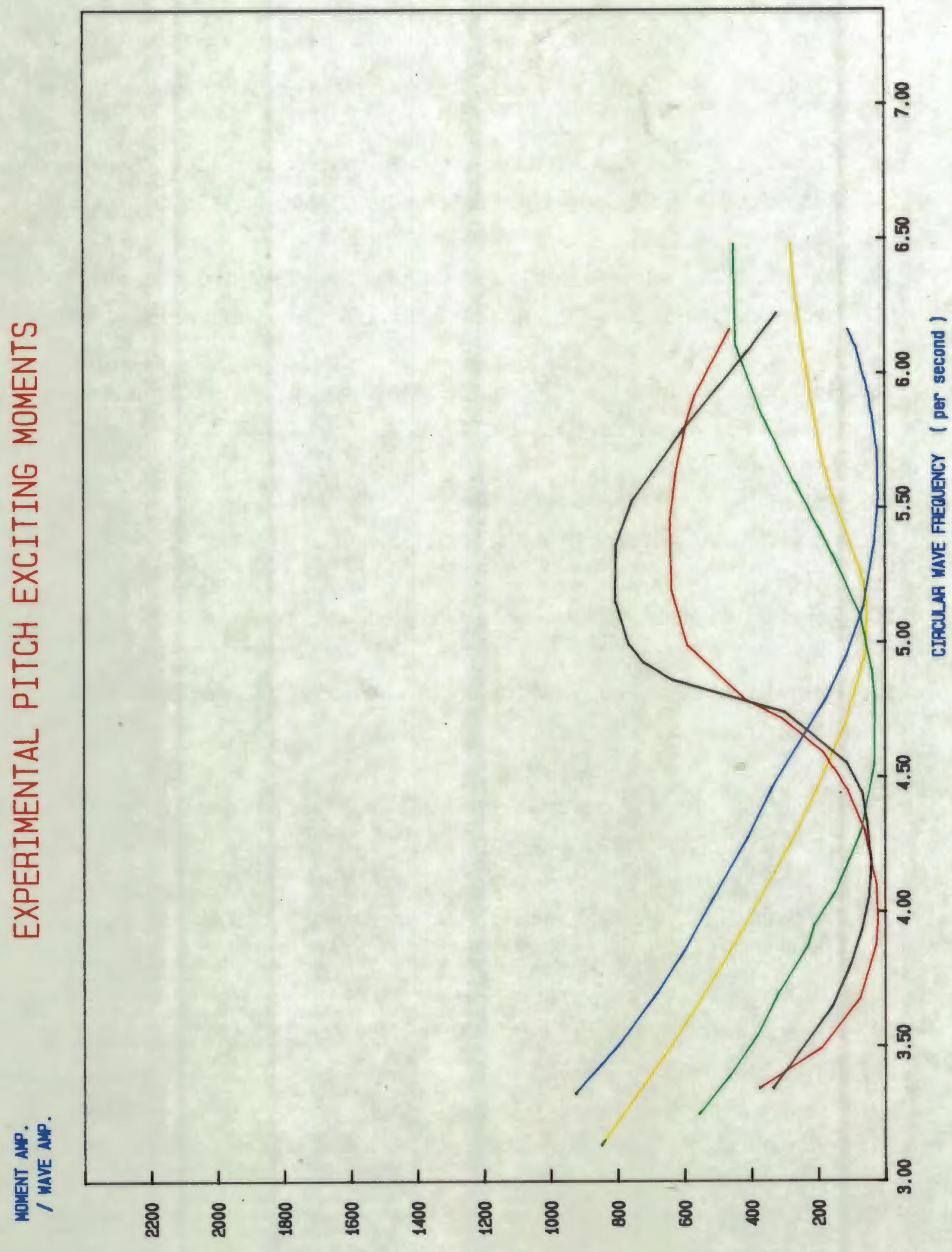

EXPERIMENTAL PITCH EXCITING MOMENTS
MOMENT AMP.
/ WAVE AMP.
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
3.00
3.50
4.00
4.50
5.00
5.50
6.00
6.50
7.00
CIRCULAR WAVE FREQUENCY (per second)

FIGURE 6.4

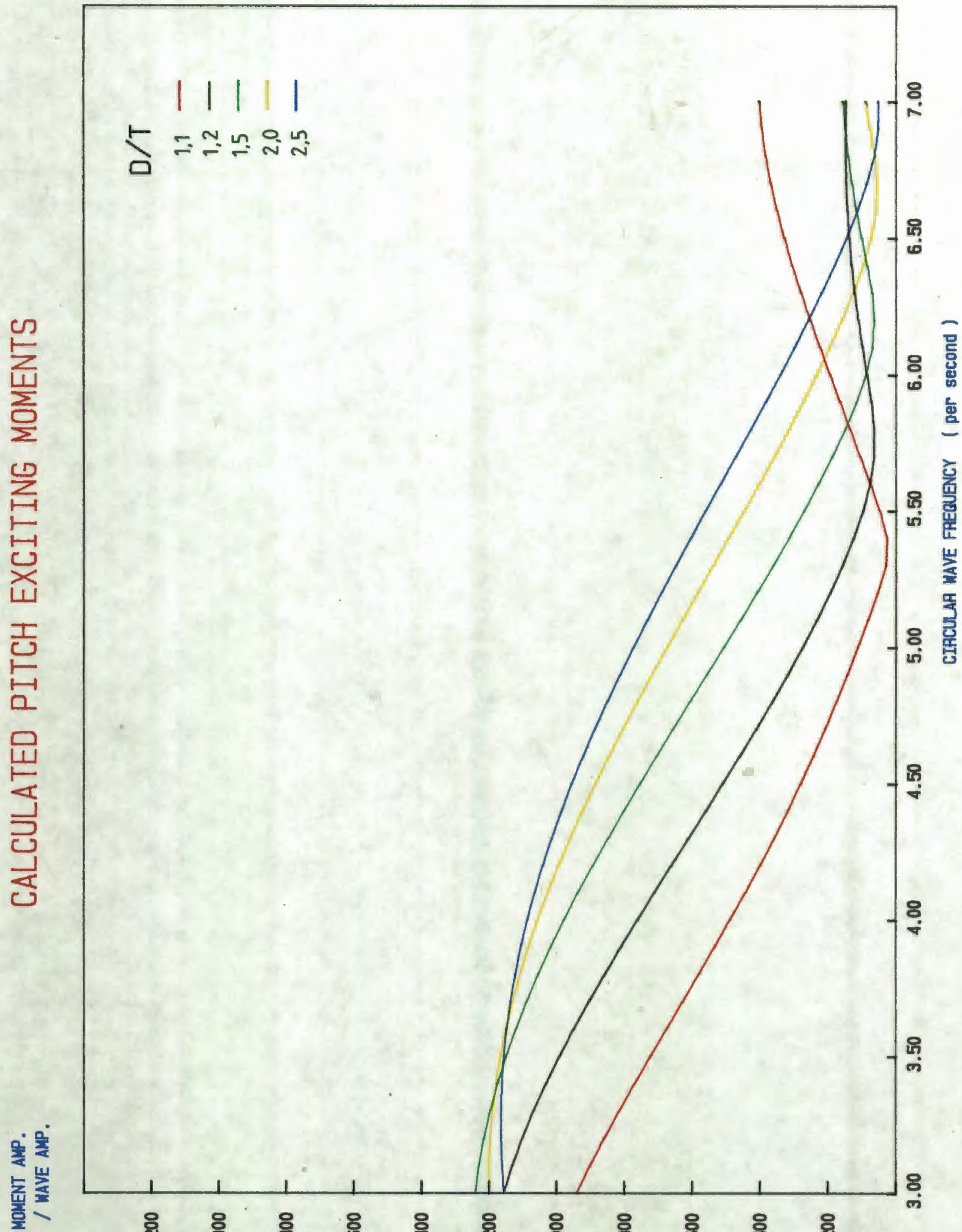
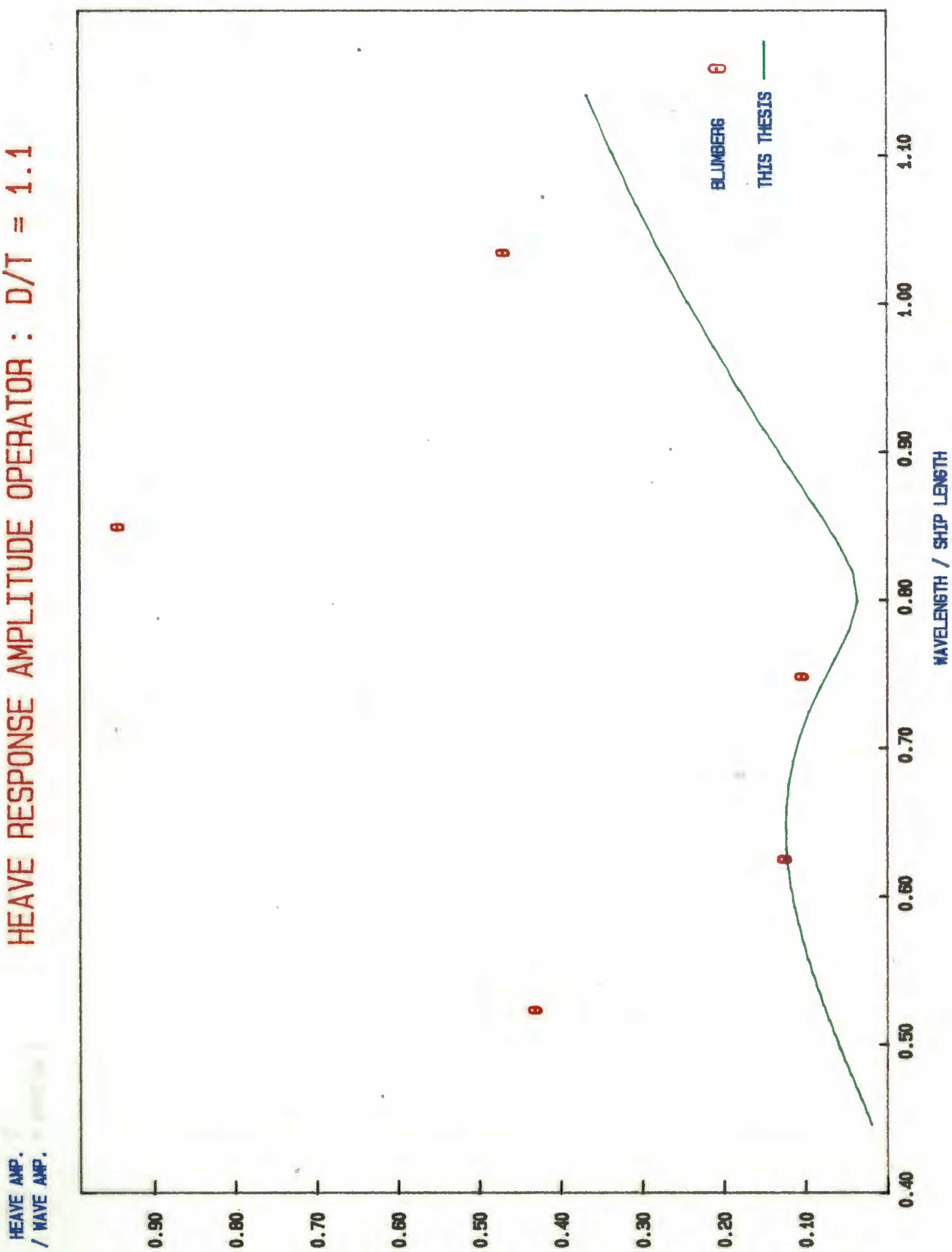

CALCULATED PITCH EXCITING MOMENTS
MOMENT AMP.
/ WAVE AMP.
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
3.00
3.50
4.00
4.50
5.00
5.50
6.00
6.50
7.00
CIRCULAR WAVE FREQUENCY (per second)
D/T
1,1
1,2
1,5
2,0
2,5

FIGURE 6.5


HEAVE RESPONSE AMPLITUDE OPERATOR : D/T = 1.1
HEAVE AMP.
/ WAVE AMP.
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.10
WAVELENGTH / SHIP LENGTH
BLUMBERG
THIS THESIS

<u>FIGURE 6.6</u>

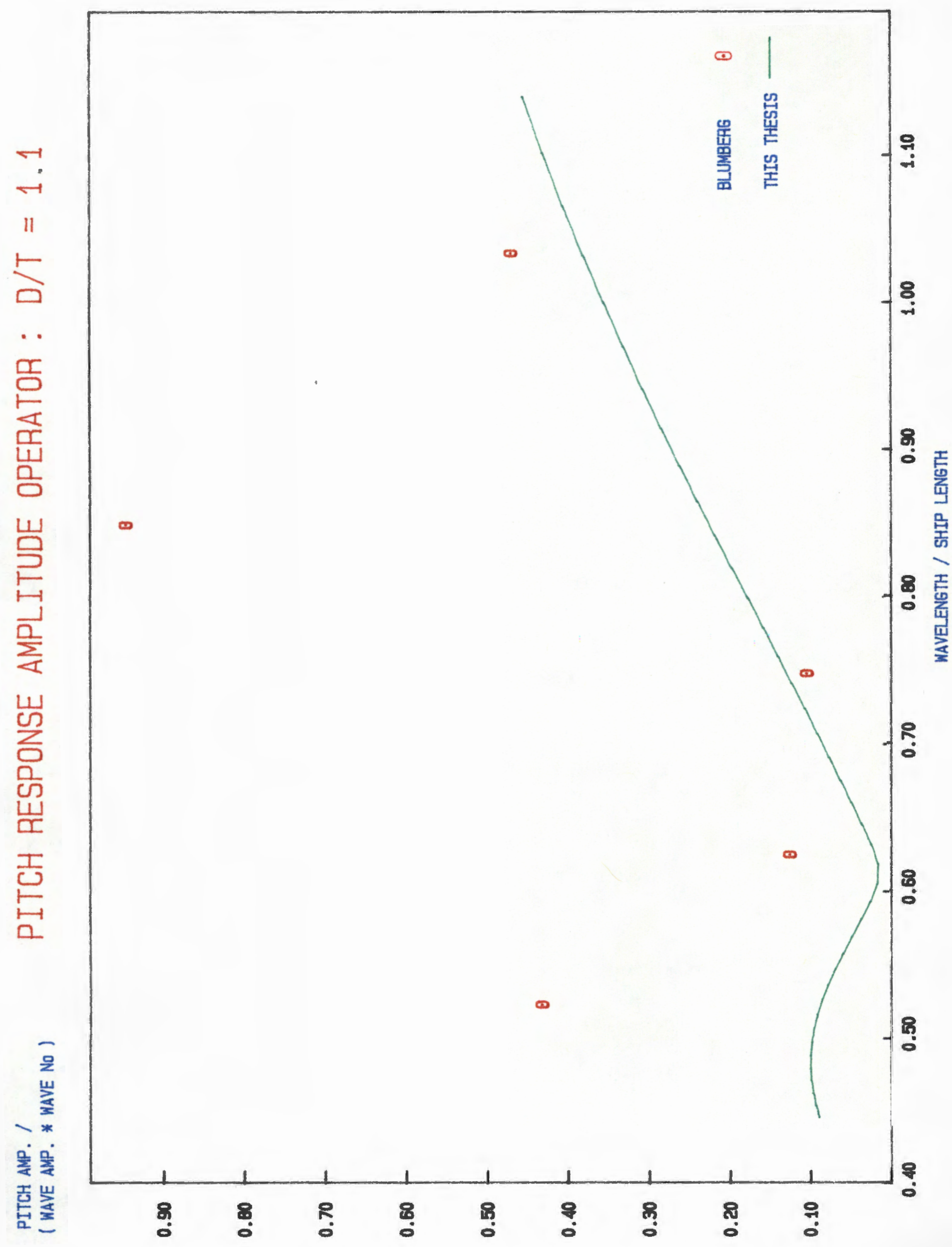

PITCH RESPONSE AMPLITUDE OPERATOR : D/T = 1.1
PITCH AMP. /
(WAVE AMP. * WAVE No)
WAVELENGTH / SHIP LENGTH
BLUMBERG
THIS THESIS
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.10

FIGURE 6.7

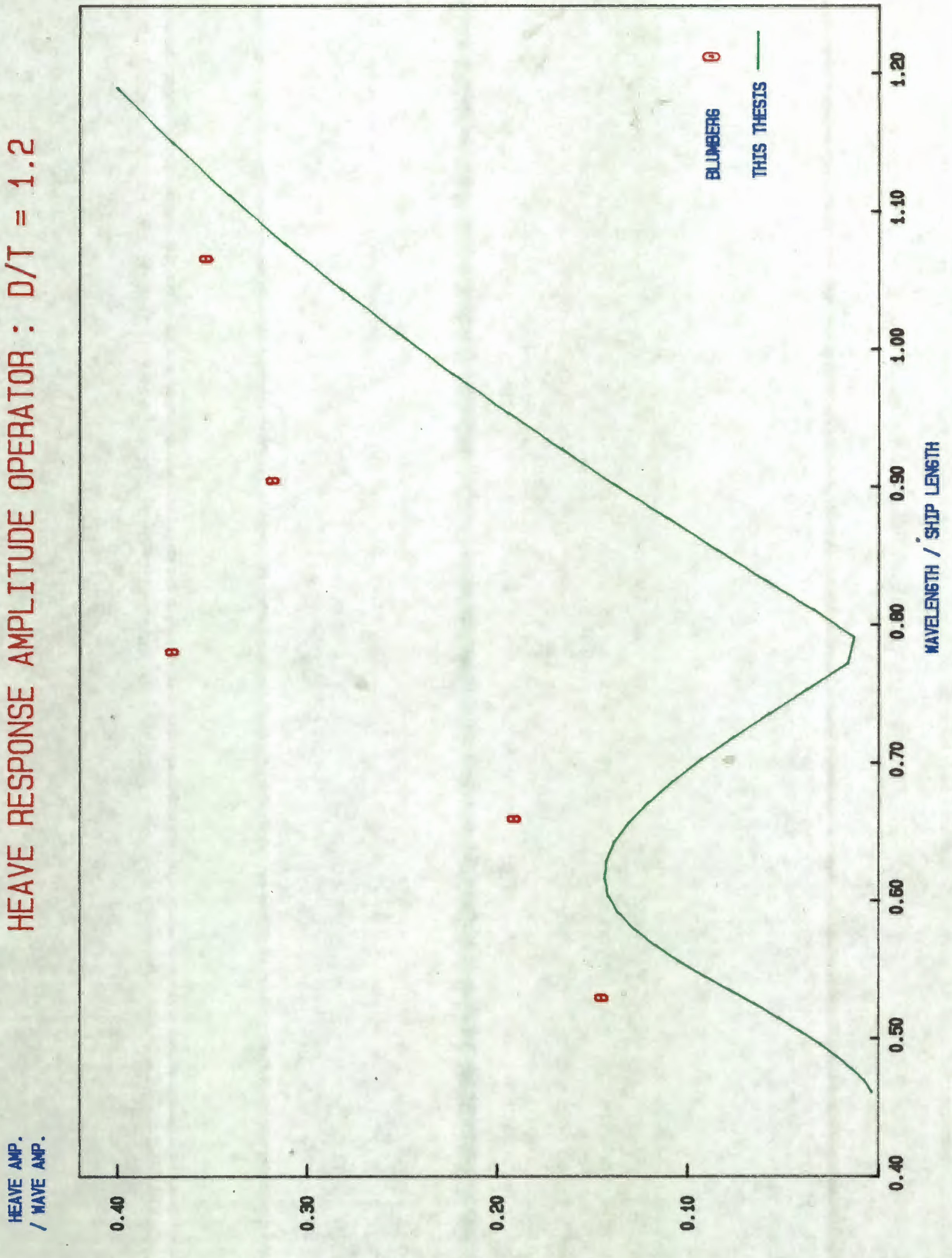

HEAVE RESPONSE AMPLITUDE OPERATOR : D/T = 1.2
HEAVE AMP.
/ WAVE AMP.
0.40
0.30
0.20
0.10
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
WAVELENGTH / SHIP LENGTH
BLUMBERG
THIS THESIS

FIGURE 6.8

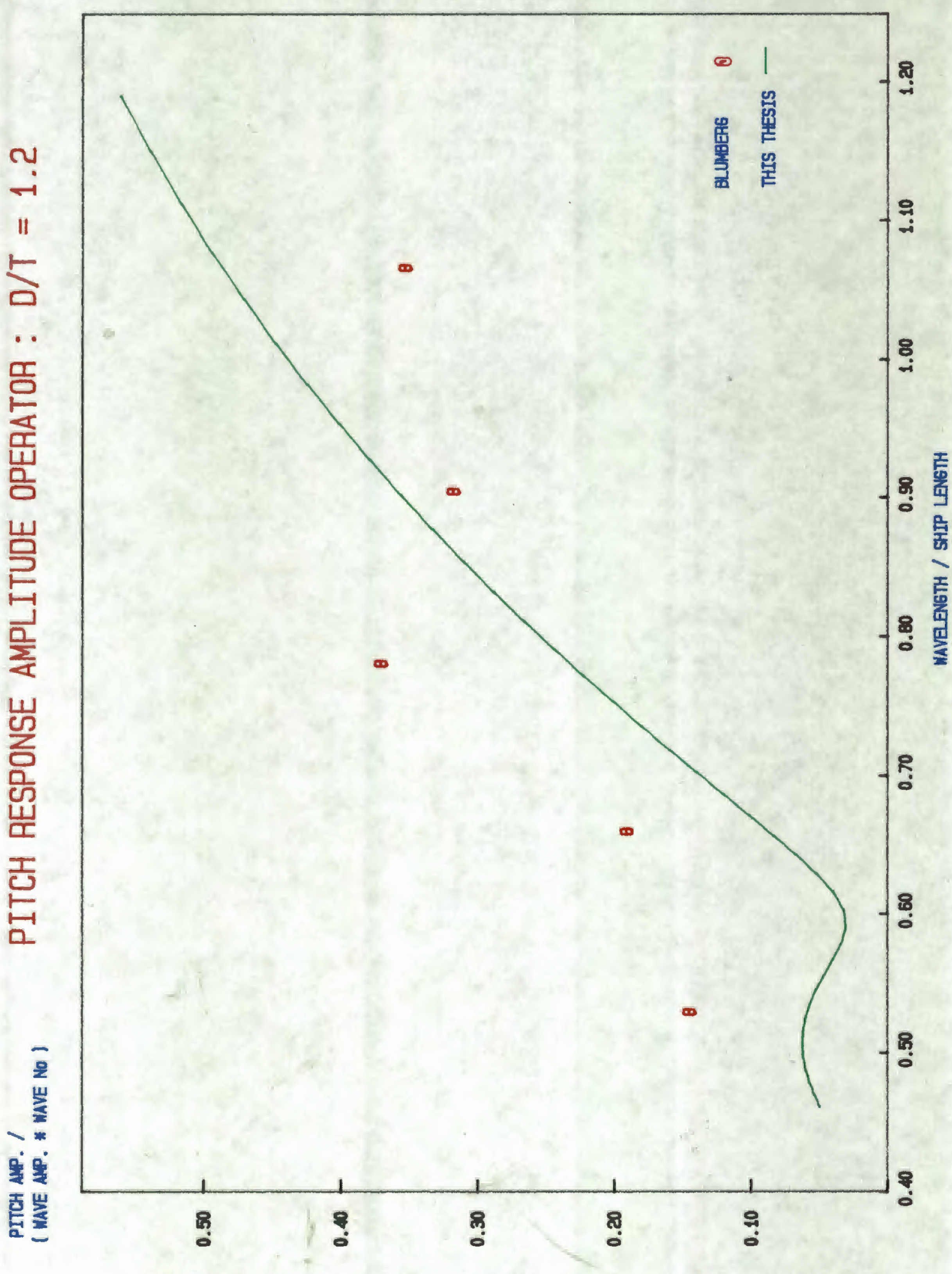

PITCH RESPONSE AMPLITUDE OPERATOR : D/T = 1.2
PITCH AMP. /
(WAVE AMP. * WAVE No)
0.50
0.40
0.30
0.20
0.10
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
WAVELENGTH / SHIP LENGTH
BLUMBERG
THIS THESIS

FIGURE 6.9

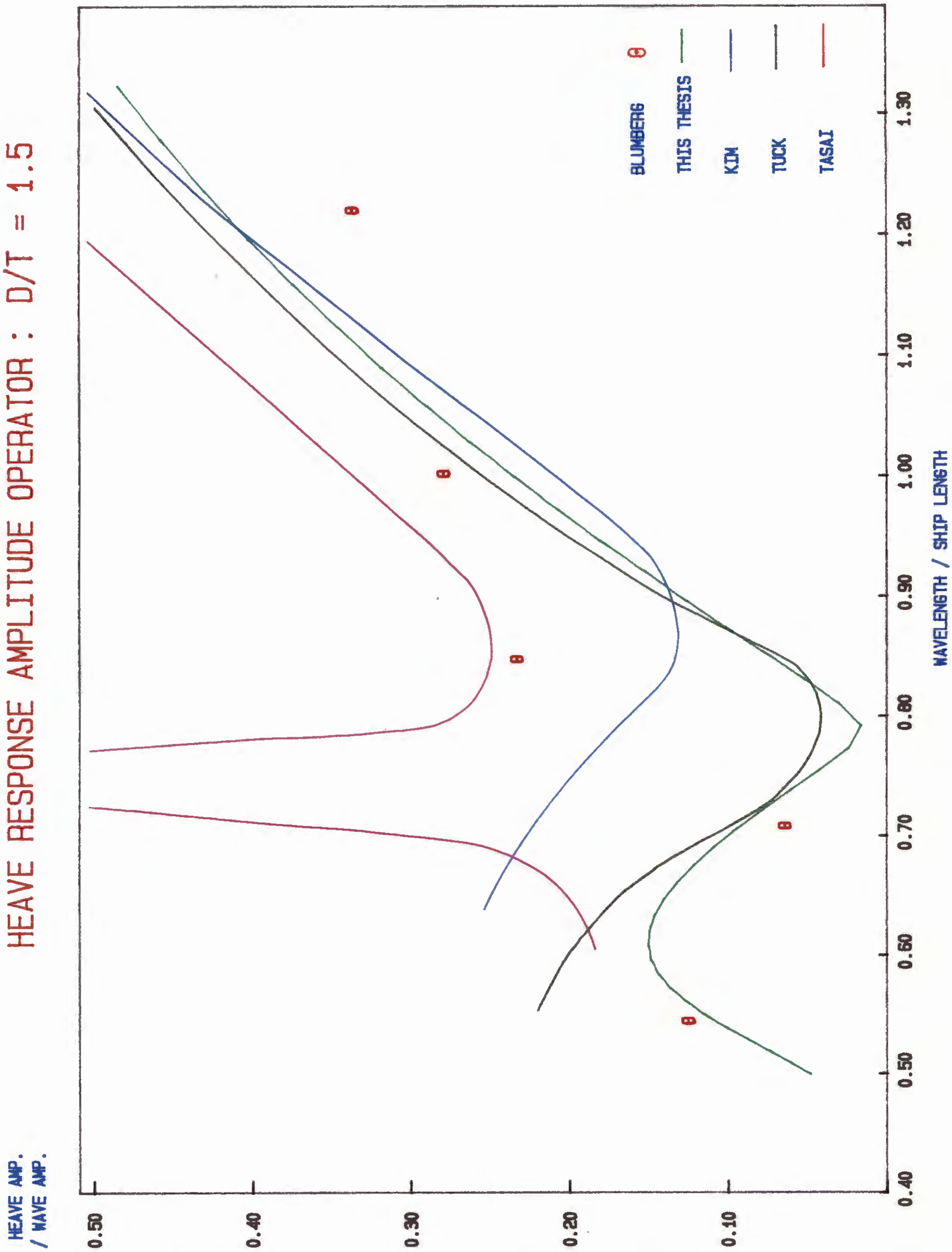
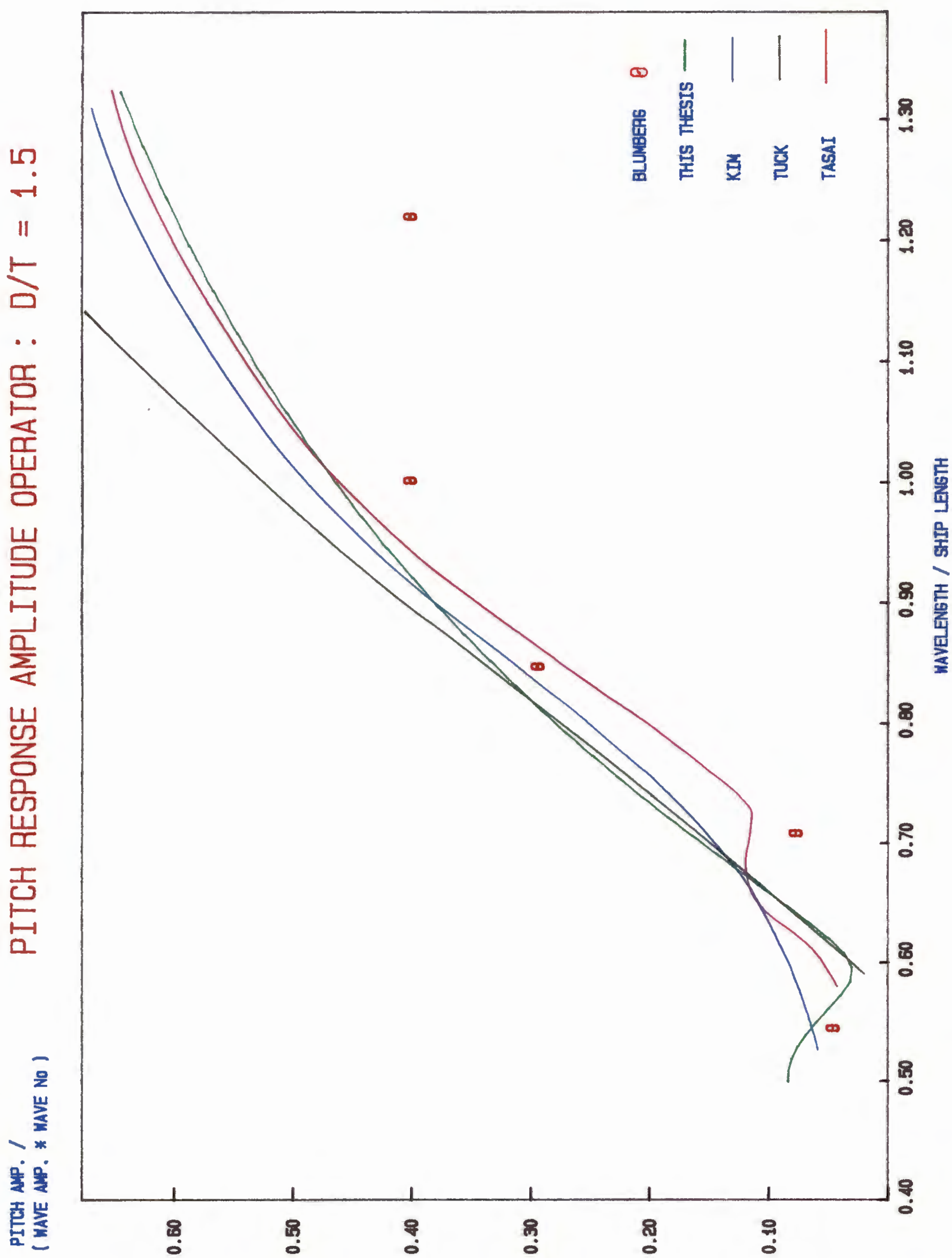

HEAVE RESPONSE AMPLITUDE OPERATOR : D/T = 1.5
HEAVE AMP.
/ WAVE AMP.
0.50
0.40
0.30
0.20
0.10
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
1.30
WAVELENGTH / SHIP LENGTH
BLUMBERG
THIS THESIS
KIM
TUCK
TASAI

FIGURE 6.1Ø



PITCH RESPONSE AMPLITUDE OPERATOR : D/T = 1.5
PITCH AMP. /
(WAVE AMP. * WAVE No)
0.60
0.50
0.40
0.30
0.20
0.10
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
1.30
WAVELENGTH / SHIP LENGTH
BLUMBERG
THIS THESIS
KIM
TUCK
TASAI

FIGURE 6.11

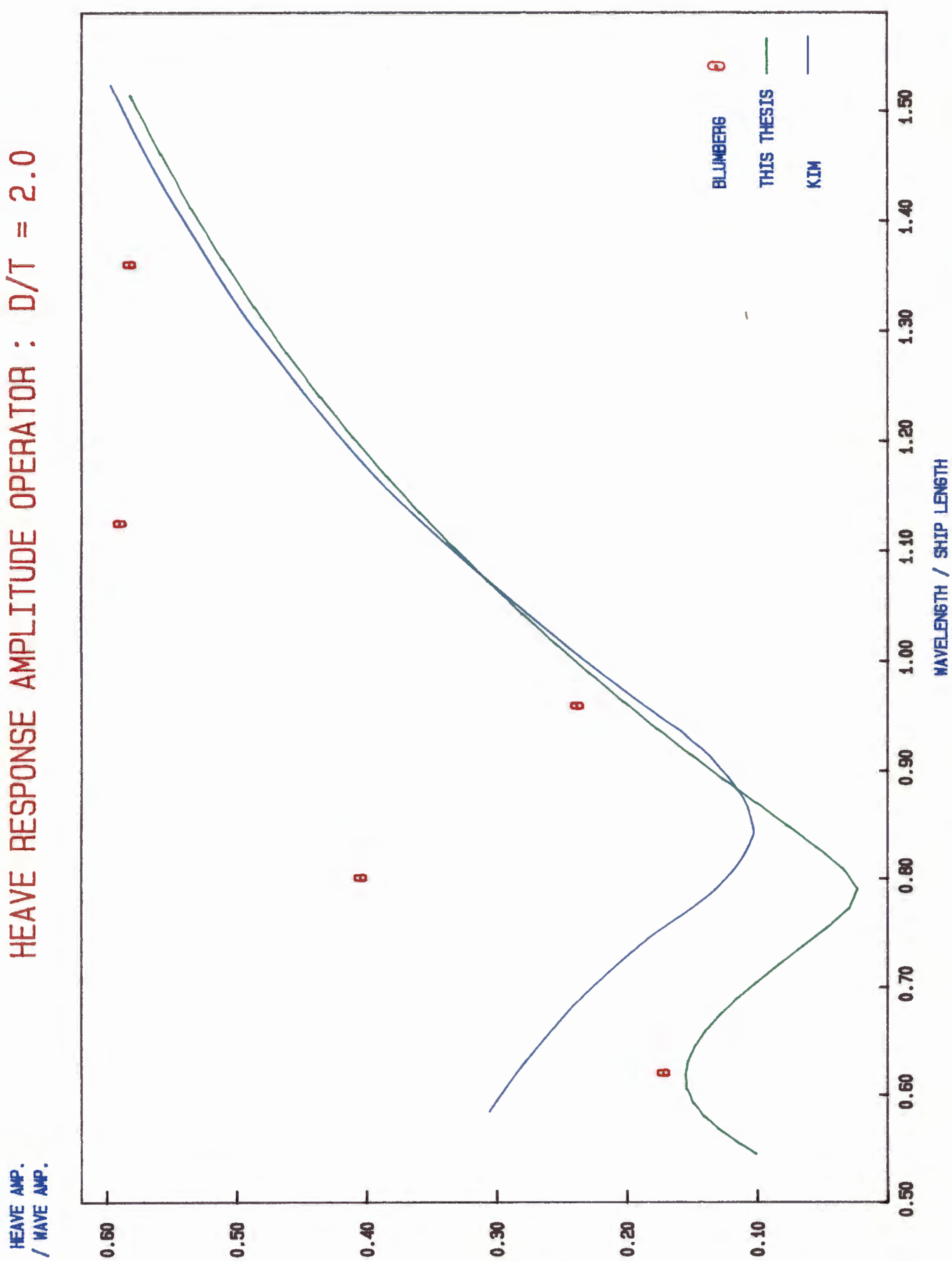

HEAVE RESPONSE AMPLITUDE OPERATOR : D/T = 2.0
HEAVE AMP.
/ WAVE AMP.
0.60
0.50
0.40
0.30
0.20
0.10
0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
1.30
1.40
1.50
WAVELENGTH / SHIP LENGTH
BLUMBERG
THIS THESIS
KIM

FIGURE 6.12

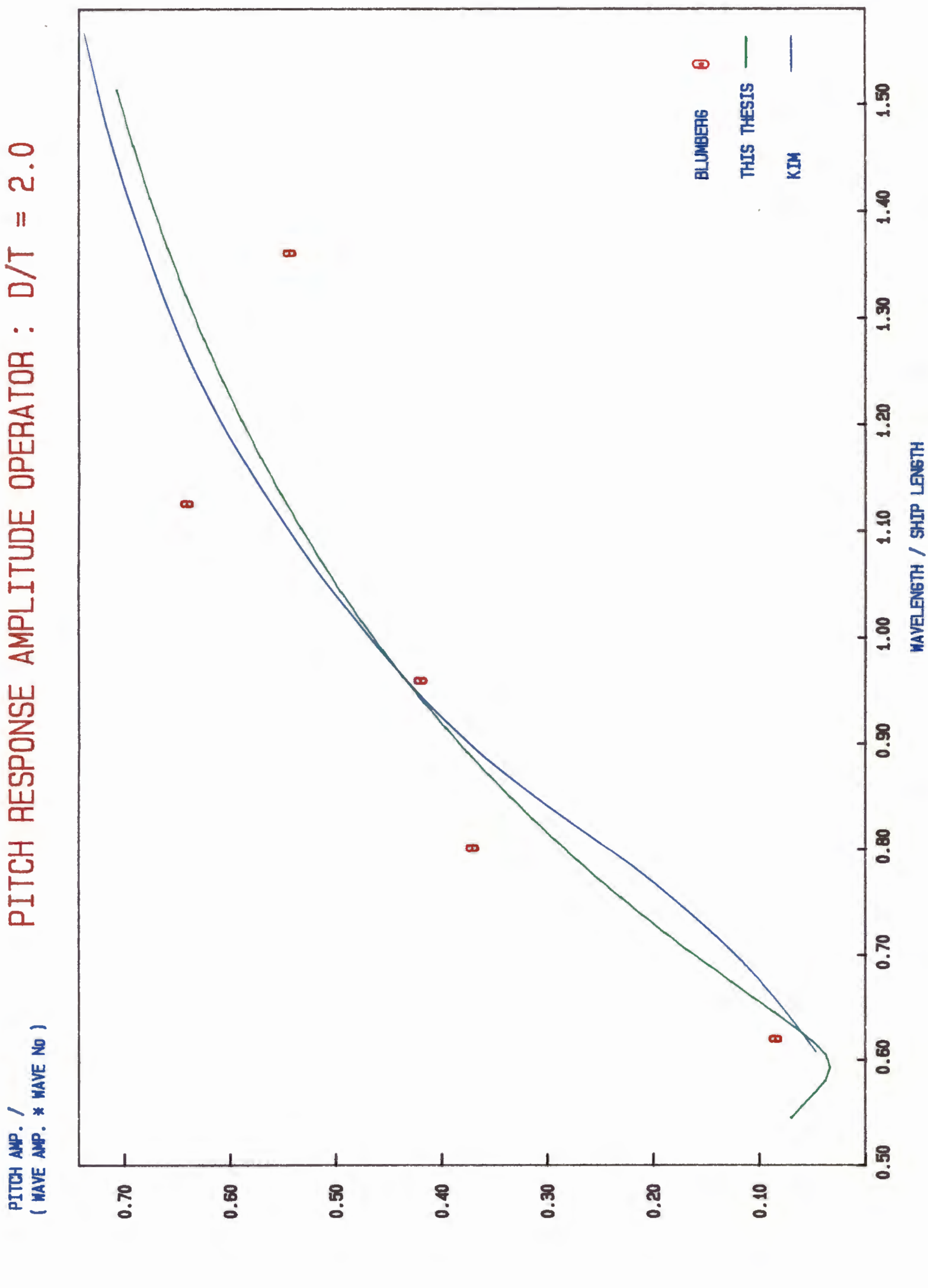

PITCH RESPONSE AMPLITUDE OPERATOR : D/T = 2.0
PITCH AMP. /
(WAVE AMP. * WAVE No)
WAVELENGTH / SHIP LENGTH
BLUMBERG
THIS THESIS
KIM
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
1.30
1.40
1.50

FIGURE 6.13

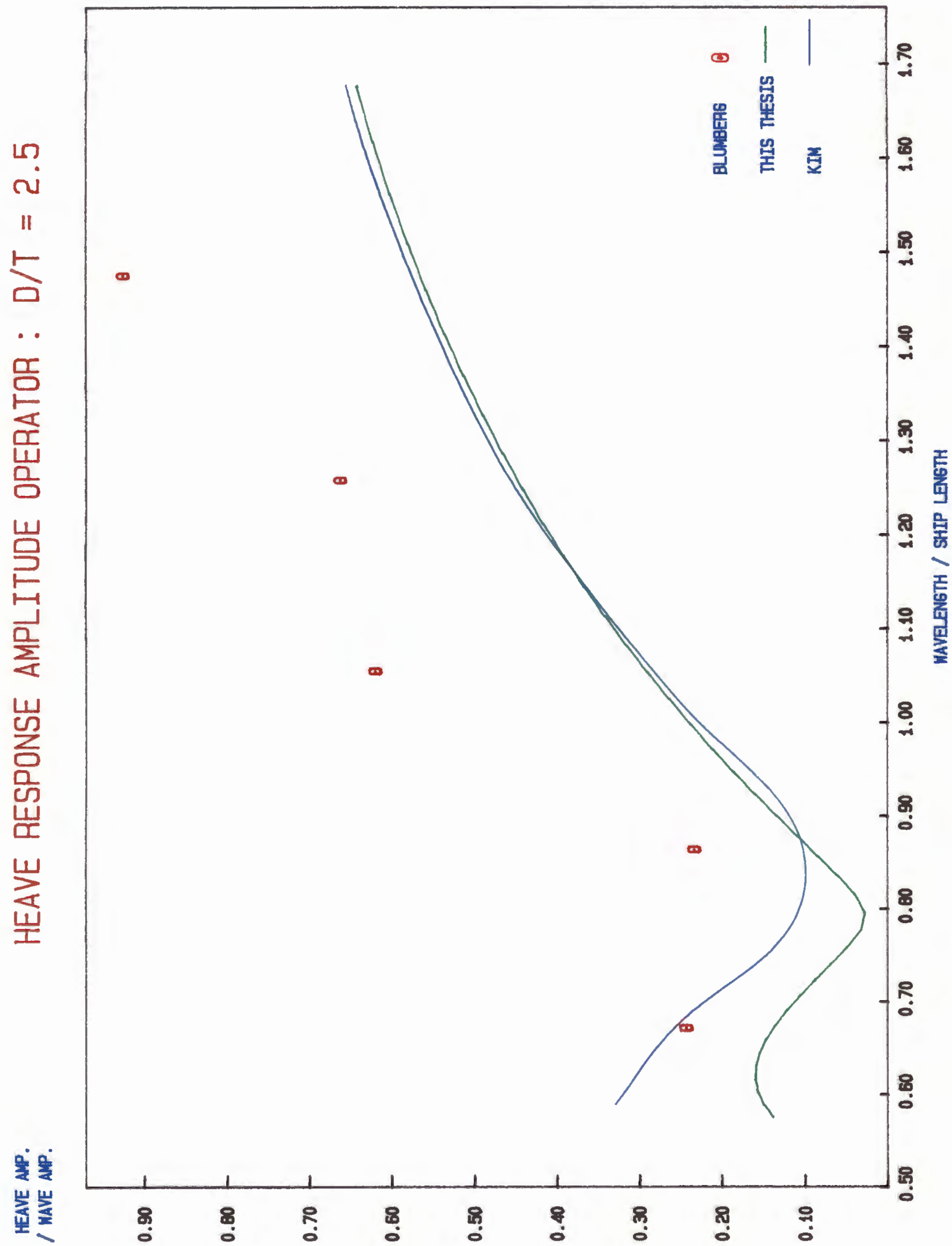

HEAVE RESPONSE AMPLITUDE OPERATOR : D/T = 2.5
HEAVE AMP.
/ WAVE AMP.
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
1.30
1.40
1.50
1.60
1.70
WAVELENGTH / SHIP LENGTH
BLUMBERG
THIS THESIS
KIM

FIGURE 6.14

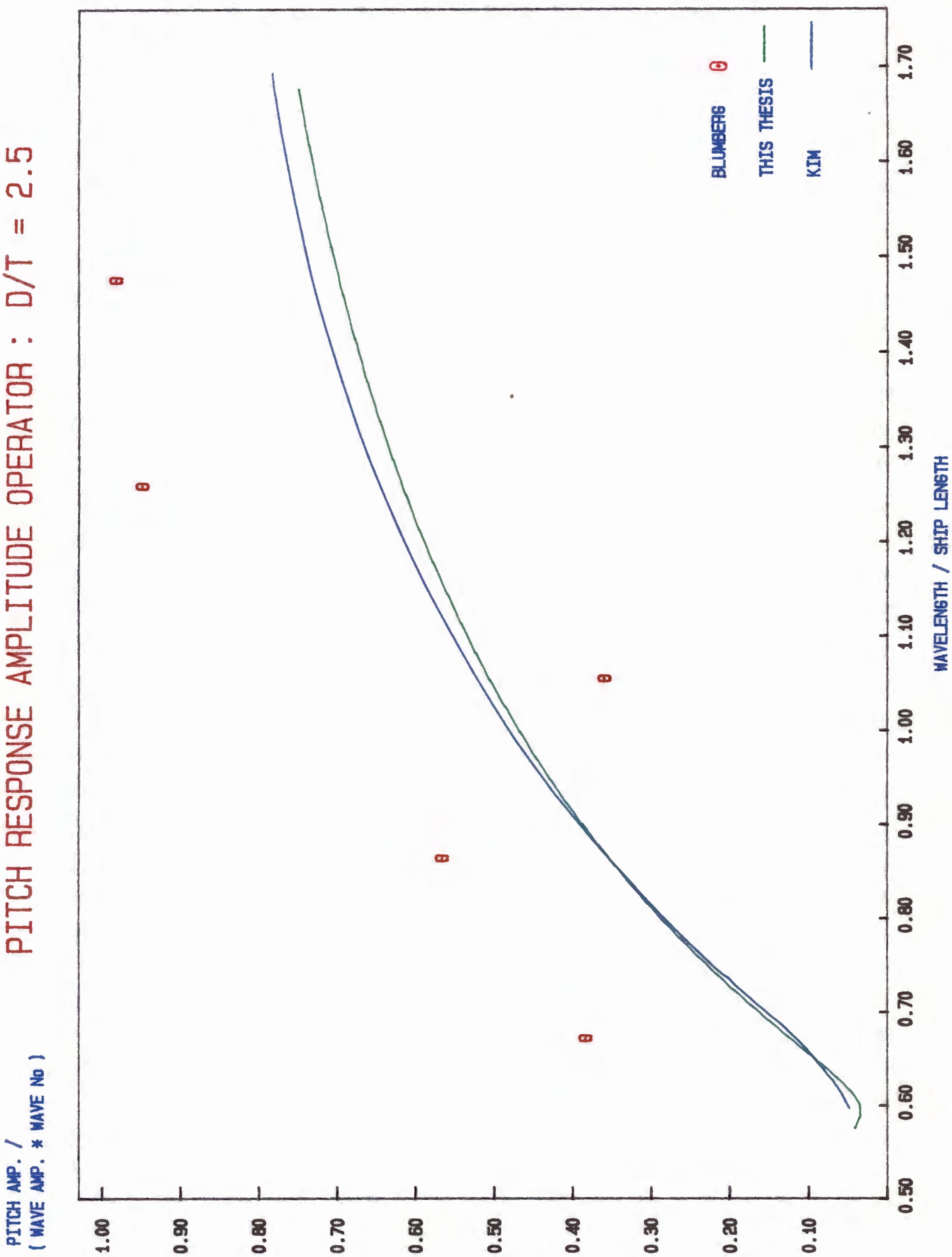

PITCH RESPONSE AMPLITUDE OPERATOR : D/T = 2.5
PITCH AMP. /
(WAVE AMP. * WAVE No)
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
1.30
1.40
1.50
1.60
1.70
WAVELENGTH / SHIP LENGTH
BLUMBERG
THIS THESIS
KIM

FIGURE 6.15

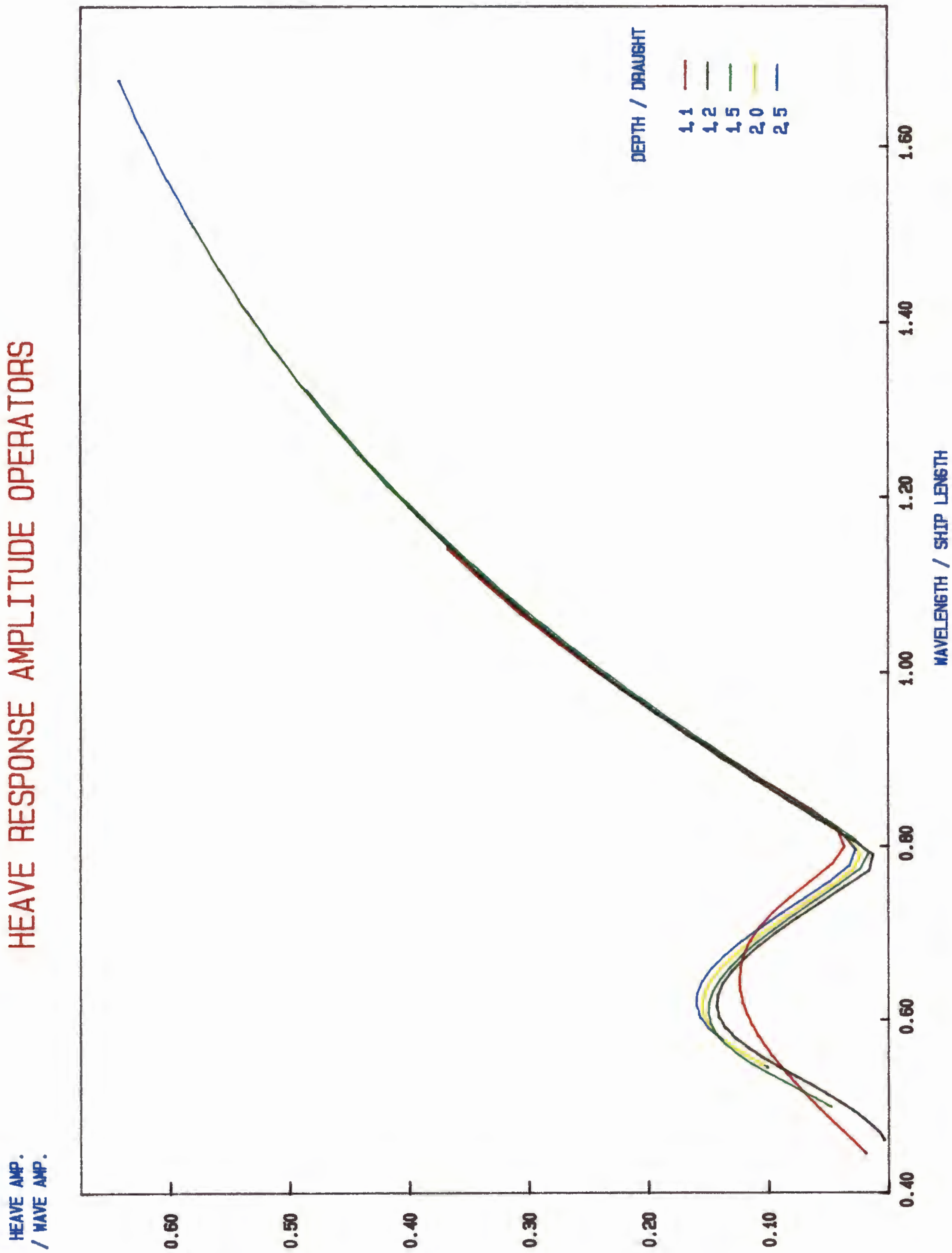
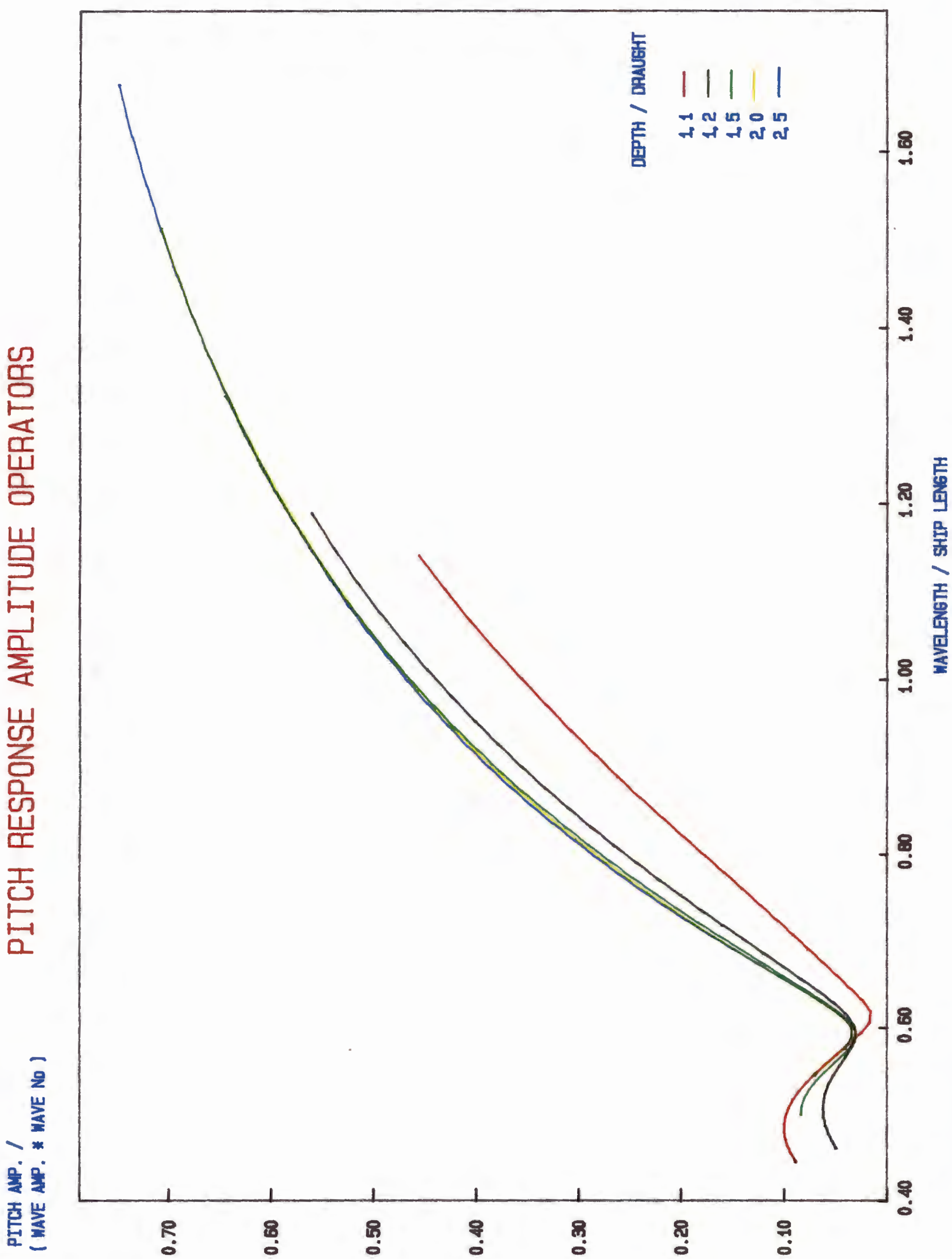

HEAVE RESPONSE AMPLITUDE OPERATORS
HEAVE AMP.
/ WAVE AMP.
0.60
0.50
0.40
0.30
0.20
0.10
0.40
0.60
0.80
1.00
1.20
1.40
1.60
WAVELENGTH / SHIP LENGTH
DEPTH / DRAUGHT
1.1
1.2
1.5
2.0
2.5

FIGURE 6.16


PITCH RESPONSE AMPLITUDE OPERATORS
PITCH AMP. /
(WAVE AMP. * WAVE No)
WAVELENGTH / SHIP LENGTH
DEPTH / DRAUGHT
1.1
1.2
1.5
2.0
2.5
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.40
0.60
0.80
1.00
1.20
1.40
1.60

7 DISCUSSION AND CONCLUSIONS

7.1 DISCUSSION OF RESULTS

7.1.1 EXCITING FORCES AND MOMENTS

On comparing the experimental exciting forces with the calculated, (figures 6.1, 6.2, 6.3 and 6.4), it can be seen that for long period waves the calculated results are in the region of twice the value of the experimental results. This is consistent with the reasoning given in chapter six. As the wave period decreases, (wave frequency increases), the two sets of data converge to a common value. This can be explained by the fact that as the wave period decreases, the effect of each particular wave on the ship is reduced. At the hypothetical limit the ship would remain motionless when acted upon by waves of almost zero period and in this instance the two sets of forces would be the same.

Looking at the calculated forces used in the final analysis, (figure 6.2), these are larger for longer period waves and tend towards relatively small values for short period waves. This is to be expected if one considers that for very long period waves the ship is moved bodily with the wave, whilst for very short waves, the ship may not move at all. In the case of pitch exciting moments, (figure 6.4), a maximum moment exists, due to the fact that the pitching motions of the ship tend to zero again as the wave approximates that of a tidal range. The start of this maximum can be seen on the extreme left of the exciting moment graph, figure 6.4.

7.1.2 MOTION RESPONSE AMPLITUDE OPERATORS

As described in chapter two, there are two major theories for the analysis of ship motions, the first being the strip theory, valid for wavelengths of the order of the ship's lateral dimensions. Looking at the wavelengths used in this thesis, see table 5.5B, this theory is not strictly valid for these waves, although according to Tuck (1978); "experience with this theory does suggest that it may remain valid even when the wavelength does approach the order of the ship length, where longitudinal interference effects ought really to be significant."

The second theory is the slender body theory derived by Tuck (1970) and applicable when the wavelength is comparable with the ship length. Examples of the predicted motion amplitudes from both of these theories are given for the depth to draught ratio of 1,5 in figures 6.9 for heave, and 6.1Ø for pitch. Here it can be seen how the slender body theory underestimates the motion responses for waves of short wavelength, particularly in the case of heave. Similarly the strip theory underestimates the motion responses for waves of long wavelength.

For the water depth to draught ratios of 1,1 and 1,2, no comparisons were available other than those of Blumberg (1983). To obtain the experimental data Blumberg measured the actual motion amplitudes of a model ship subjected to waves, in the wave channel decribed in section 4.1.2.

Looking at the two heave response amplitude operators for these depths, figures 6.5 and 6.7, it can be seen that the results of Blumberg straddle those of this thesis, however Blumberg tends to predict greater motion amplitudes for longer wavelengths. A similar pattern exists for the case of the pitch response amplitude operator of depth to draught ratio 1,1, figure 6.6. At the depth to draught ratio of 1,2, (figure 6.8), there is a marked change in the trend of

Blumberg's results. Instead of showing large oscillations between successive points as before, the points lie on a smooth curve. The results of the present thesis bear a slight resemblance to this curve with the exception of a local minimum at the wavelength to ship length ratio of 0,60. The agreement between the two sets of data is best in the region where the wavelength is of the order of the ship length, which is also the slender body theory domain. (Recall that the formulae used for the exciting forces in this thesis were drawn from the strip theory.)

The next water depth investigated was the depth to draught ratio of 1,5, which is the most frequently documented case in literature. Superimposed on the heave ROA, (response amplitude operator), figure 6.9, are the theoretical results of Kim, (using the strip theory), Tuck, (using the slender body theory), Tasai's experimental results and Blumberg's experimental results. The Tasai results are interpolated from depth to draught ratios of 1,3 and 1,8, the 1,5 case not having been investigated. The results of the present thesis tend to lie between the theoretical results of Kim and Tuck, with the exception of wavelengths less than half the ship length, where both sets of theoretical results predict larger motion amplitudes than this thesis. The empirical results of Tasai are consistently 2Ø% to 25% higher than those of Kim, Tuck and this thesis. These results are also dominated by a resonance condition at wavelengths of 75% of the ship length, which although not shown in the RAO, (figure 6.9), peaks at a heave amplitude to wave amplitude ratio of Ø,7. The experimental results of Blumberg tend to agree best with other author's results at wavelengths of the order of the ship length. For longer wavelengths, the theory predicts larger motion amplitudes than found by Blumberg, but smaller motions than found by Tasai. At wavelengths of less than half the ship length, both Tasai's and the results of this thesis converge on a downward trend, as can be expected for short period waves.

The differences between the four sets of data, Kim, Tuck, Tasai and this thesis are much smaller in the case of pitch motions for the 1,5 depth to draught ratio, figure 6.1Ø. Tuck's slender body theory gives slightly larger motion amplitudes at wavelengths greater than the ship length. Blumberg's data only coincide with the other authors' results up to wavelengths of about 95% the ship length. There after Blumberg predicts much smaller motion amplitudes than other authors.

For the heave RAO's at the depth to draught ratios of 2,0 and 2,5, (figures 6.11 and 6.13), the experimental results of Blumberg differ greatly from both the results of this thesis and those of Kim. The predicted motion amplitudes of this thesis follow those of Kim closely down to wavelengths of 90% of the ship length. For shorter wave lengths, the strip theory domain, Kim predicts greater amplitudes of motion than this thesis. Blumberg's results are generally greater than those of this thesis, not showing signs of following any pattern. For the pitch RAO at the depth to draught ratio of 2,0, (figure 6.12), the results of this thesis follow closely the theoretical results of Kim, deviating as for heave at the smaller wavelength range. However in this case, unlike heave, this thesis predicts greater motion amplitudes than Kim. Blumberg's results tend to be closer to those of this thesis than for the corresponding heave case, varying around the theoretical results of Kim. As the depth to draught ratio increases to 2,5, (figure 6.14), the differences between Kim's theoretical results and the results of this thesis become minimal, Kim predicting slightly larger pitch amplitudes for wavelengths greater than the length of the ship. For the case of the wavelength close to the ship length, Blumberg's results are smaller than those of this thesis. With the exception of this case, the rest of Blumberg's results are significantly larger than those of this thesis or the theoretical results of Kim.

Looking at the five heave RAO's of this thesis superimposed onto one graph, figure 6.15, one can see the general trends of a local minimum at wavelengths of 80% of the ship length and a similar maximum at wavelengths of 60% of the ship length. As the water depth becomes very shallow, the amplitude of the local maximum increases rapidly, pulling the local minimum up with it. It is of interest to note that for long waves, 1,7 times the ship length, the maximum motion amplitude predicted is only 60% of the wave height.

The five pitch RAO's, figure 6.16, follow a similar trend to the heave RAO's, with the local minimum at 60% of the ship length and the local maximum of about 50% of the ship length. In this case where the water depth becomes very shallow, the local maxmimum increases in amplitude, while the local minimum decreases.

7.2 CONCLUSIONS

On looking at the added mass, added moment of inertia and damping coefficient with respect to water depth, figures 5.2B and 5.3B, it can be seen that as predicted by Kim (1968), the coefficients increase rapidly with decrease in water depth. In general it would appear that when moving from a depth to draught ratio of 2,5 down to 1,1, the damping coefficients for both heave and pitch double, the added moment of inertia triples and the added mass quadruples.

By finding experimentally the various coefficients required for the equations of motion, this thesis was also a check on the theoretical formulae for the various coefficients. Unfortunately the exciting force equations could not be checked in a similar fashion, as explained in chapter six. However the experimental data were used in obtaining these forces as well. The relatively good agreement between the empirical results of this thesis and results of other authors would indicate that the theoretical formulae do represent the coefficients which they are claimed to approximate, for at least the case of zero or low forward speeds.

On comparison of the results of this thesis with those of Blumberg, the agreement was not good at all. Blumberg predicts motion amplitudes that are generally greater than those of this thesis and occasionally show some oscillation or resonance conditions. One explanation for this disparity of results could be the fact that Blumberg was not working in "open water". Open water conditions being defined, (Barrass 1979), as

$$\text{minimum channel width} = B.(\ 7,7 + 45.(1-C_W)^2)$$

where C_W = water line plane area coefficient

B = extreme breadth

For the model used by Blumberg the minimum width necessary to simulate unhindered open water conditions is 2,916 m. The width of the wave channel used for the experiments was 1,524 m or 52% of the required width. Thus a confined channel situation was modelled and hence it is not surprising that the results of Blumberg do not correlate well with the results of this thesis or other authors working in open water conditions.

REFERENCES AND BIBLIOGRAPHY

Andersen, P., 1979
"Ship motions and sea loads in restricted water depths", Ocean Engng 6, pp.557-569.

Bai, K.J., and Yeung, R.W., 1974
"Numerical solutions to free surface flow problems", Proc. 10 ONR Symp.

Barrass, C.B., 1979
"The phenomenon of ship squat", Terra at Aqua,18.

Barnaby, K.C., 1969
"Basic naval architecture", 6th ed., Hutchinson and Co., London pp.15-26.

Beck, R.F. and Tuck, E.O., November 1971
"Heave and pitch of ships in shallow water", 4th Australian Conf. on Hydraulics and Fluid Mechanics, Monash Univ., Australia.

Beck, R.F., Newman, J.N., and Tuck, E.O., September 1975
"Hydrodynamic forces on ships in dredged channels", J. Ship Res., 19, No.3, pp.166-171.

Berhault, C., 1980
"An integro-variational method for interior and exterior free surface flow problems", Appl. Ocean Res. Vol. 2, No 1, pp.33-38

Beukelman, W. and Gerritsma J., March 1982
"De verdeling van de hydrodynamische massa en demping over een in ondiep water oscillerend scheepsmodel", Technische Hogeschool Delft, Laboratorium voor Scheepshydromechanica, Rap.nr.546A.

Bickford, J.H., 1972
"Mechanisms for intermittent motion", Industrial Press Inc., New York.

Bishop, R.E.D. and Price, W.G., 1974
"Dynamics of marine vehicles and structures in waves", IMECHE, Editors.

Bishop, R.E.D., Price, W.G. and Tam, P.K.Y., April 1977
"A unified dynamic analysis of ship response to waves", Trans. RINA, Vol. 3

Blagoveschensky, S.N., 1962
"Theory of ship motions", Dover Publications, New York, chs I,V,IX.

Blumberg, G.P., September 1983
"The vertical motions of a model ship", Unpublished MSc Thesis, Univ. Cape Town.

Brennen, C.E., January 1982
"A review of added mass and fluid inertial forces", Naval Eng. Lab., Port Hueneme, California.

Buhr Hansen, J. and Svendsen, I.A., June 1974
"Laboratory generation of waves of constant form", Proc. 14th ICCE, Ch.17, pp.321-344.

Chakrabarti, S.K., August 1980
"Laboratory generated waves and wave theories", J. Waterw. Port Coastal Ocean Div., Proc. ASCE,106, No.WW3.

Chen, H.S., and Mei, C.C., 1974
"Oscillations and wave forces in a man-made harbor in the open sea", 10 Symp. Naval Hydrodynamics, Cambridge, Mass.

Coldwell, T.G., September 1983
"Marine traffic behaviour in restricted waters", J. of Navigation, Vol 36, No 3.

Constantine, T., May 1960
"On the movement of ships in restricted waterways", J. Fluid Mech., 9, Pt.2, pp.247-256.

Daubert, A., 1970
"Quelques considerations sur les differentes simulations et methodes de calcul du comportement des structures immergees ou flottantes en mer", La Houille Blanche, No 2.

Den Hartog, J.P., 1934
"Mechanical vibrations", First Ed., McGraw-Hill Book Co., New York.

Durbin, T.E., June 1977
"Net bottom clearance and shallow water effects on manoeuvrability", U.S. Coast Guard, Mem G-WS/73.

Edstrand, H. and Norrbin, N.H., July 1978
"Shallow water phenomena and scale model research - some first experience from the SSPA Maritime Dynamics Laboratory", Swedish State Shipbuilding Prog., No.81.

Faltinsen, O.M., March 1973
"A numerical investigation of the Ogilvie-Tuck formulas for added-mass and damping coefficients", J. Ship Res., 18, No.2, pp.73-84.

Flick, R.E. and Guza, R.T., February 1980
"Paddle generated waves in laboratory channels", Proc. ASCE, 106, No.WW1, pp.79-90.

Freberg, C.R. and Kemler, E.N., February 1949
John Wiley and Sons, Inc., Second Edition, New York.

Garisson, C.J., and Chow, P.W., 1972
ASCE Waterways and Harbours Div. 98, pp.375-392.

Garrett, C.J.R., March 1971
"Wave forces on a circular dock", J. Fluid Mech. 46(1) 129.

Gerritsma, J. and Beukelman, W., 1966
"Comparison of the calculated and measured heaving and pitching motions of a Series 60 Cb = 0,7 ship model in regular longitudinal head waves", Technische Hogeschool Delft, Laboratorium voor Scheepsbouwkunde, Rep. No.139.

Grim, O., 1962
"Eine Methode fur eine Genauere Berechnung der Tach- und Stampf-bewegungen in glattem Wasser", HSVA, Bericht No 1217.

Guoping, M., September 1980
Chalmers Univ. Naval Arch., Div. Ship Hydromechanics, Rep. No.58, Gothenburg.

Hansen, H.M. and Chenea, P.F., 1952
"Mechanics of vibration", John Wiley and Sons, Inc., New York.

Havelock, T.H., 1942
"The damping of the heaving and pitching motions", Philosphical Mag. Vol.33, pp.666-673.

Havelock, T.H., 1958
"The effect of speed advance upon the damping of heave and pitch", Inst. Nav. Archit., Trans 100, pp.131-135.

Hooft, J.P., April 1973
"Manoeuvring large ships in shallow water", J. of Navigation, Vol 26, No 2.

Hooft, J.P., August 1974
"The behaviour of a ship in head waves at restricted water depths", Netherlands Scheepsstudiecentrum T.N.O., No.188s.

Hoppe, K.G., 1971
"Scale effects in manoeuvring tests with small ship models", ECOR Symp. on Ocean's Challenge to S.A. Engineers, Stellenbosch.

Jameson, A.H., 1937
"An introduction to fluid mechanics", Longmans, Green and Co., London.

Kim, C.H., July 1980
"A computation of the motions of a large tanker in shallow water", 19th American Towing Tank Conf., Univ. of Michigan.

Kim, C.H., 1968
"The influence of water depth on the heaving and pitching motions of a ship moving in longitudinal regular head waves", Schiffstechnik, Bd 15, Heft 79.

Koele, L.E., August 1971
"Behaviour of large tankers in shallow water in relation to the dimensions of an approach channel", Symp. on Offshore Hydrodynamics, N.S.M.B. publ. No.375, Wageningen.

Korvin Kroukovsky, B.V. and Jacobs, W.R., November 1957
"Pitching and heaving motions of a ship in regular waves", Trans. SNAME, New York.

Kreyszig, E., 1962
"Advanced engineering mathematics", John Wiley and Sons, New York.

Lackenby, H. and Milton, D., 1964
"D.T.M.B. Standard Series 60", BSRA Rep. NS69, Naval Arch. Rep. No.19.

Lewis, F.M., 1929
"The inertia of water surrounding a vibrating ship", Soc. Nav. Archit. Mar. Engng, Trans 37, pp.1-20.

Lloyds, 1971
"World Tanker Register", London.

Luttmer, R.G.J.C., October 1977
"Verticale scheepbewegingen op ondiep water", Ph.D dissertation, Technische Hogeschool Delft.

Macmillan, J.F., 1958
"Principles of Naval Architecture", SNAME, New York, Ch.IV, pp.138-146.

Madsen, P.A., Svendsen, I.A. and Michaelsen, C., 1980
"Some recent results of wave induced motions of a ship in shallow water", Proc 17th ICCE, vol 3, pp.3043-3062.

Manley, R.G., 1942
"Fundamentals of vibration study", Chapman and Hall Ltd., London.

Milne-Thomson, L.M., 1938
"Theoretical Hydrodynamics", MacMillan and Co. Ltd., London.

Muckle, W., 1975
"Naval architecture for marine engineers", Newnes-Butterworths, London, Chs.2,5,6,11.

Myklestad, N.O., 1944
"Vibration Analysis", McGraw-Hill Book Co. Inc., New York.

National Maritime Institute, 1980
"Ship behaviour in ports and their approaches", pt.1, N.P.C., London.

Newman, J.N., 1970
"Applications of slender body theory in ship hydrodynamics", J. Fluid Mech., pp.67-94.

Newman, J.N., 1978
"The theory of ship motions", Advances in App. Mech., 18, pp.221-283.

Ogilvie, T.F., March 1969
Univ. of Michigan, Dept. Naval Archit. and Marine Engng., Report No 13.

Pilkey, W.D. and Chang, P.Y., 1978
"Modern formulas for statics and dynamics", McGraw-Hill Book Co., New York.

Plotkin, A., June 1977
"Heave and pitch motions in shallow water including the effect of forward speed", J. Fluid Mech., 80, pt.3, pp.433-441.

Price, E.W.G. and Bishop, R.E.D., 1974
"Probabilistic theory of ship dynamics", Chapman and Hall, London.

Roberts, J.A., 1952
"Spring design and calculations", Tech. Res. Lab., Herbert Terry and Sons Ltd., Redditch, Sixth Ed.

Salvesen, N., Tuck, E.O. and Faltinsen, O., November 1970
"Ship motions and sea loads", Trans. SNAME, 78, pp.250-287.

Scholtz, D.J.P., September 1980
"A brief literature on wave absorbers used in harbours", NRIO memorandum 8017, Stellenbosch.

Stoker, J.J. et al, 1957
"Water waves : The motion of a ship in a seaway", Interscience, New York.

Svendsen, I.A. and Madsen, P.A., 1981
"The dynamics of wave induced ship motions", Ocean Eng., 8, No.5, pp.443-479.

Tasai, F. and Kim, C.H., May 1968
"Effect of shallow water on the natural period of heave", Res. Ins. Applied Mechanics, Kyushu Univ., 16, No.54, pp.229-233.

Tasai, F., Takaki, M. and Ohkusu, M., 1978
"Ship motions in restricted waters", Res. Ins. Applied Mech., Vol.26, No.81, pp.23-81.

Taylor, D.J., February 1982
"Ship motion studies at Richards Bay harbour", Coastal engineering course, Civ. Eng. Dept., Univ. Cape Town.

Thomas, G.B. and Finney, R.L., 1979
"Calculus and analytic geometry", Addison-Wesley Pub. Co., USA.

Thomas, M.W.S., June 1977
"Measuring the movement of a ship model", Hydraulics Research Station, Tech. memor.2/76, Wallingford.

Timman, R., and Newman, J.N., 1962
J. Ship Res. 5 pp1-7.

Tothill, J.T., January 1966
"Ships in restricted channels - a correlation of model tests, field measurements and theory", NRC Mech. Eng. Rep. MB-264, Canada.

Tuck, E.O., December 1970
"Ship motions in shallow water", J. Ship Research, 14, No.4, pp.317-328.

Tuck, E.O., and Taylor, P.J., 1970
8 Symp. on Naval Hydrodynamics, Pasadena.

Tuck, E.O., 1978
"Hydrodynamic problems of ships in restricted waters", Ann. Rev. Fluid Mech., 10, pp.33-46.

U.S. Army Corps of Engineers, 1973
"Shore protection manual", 1, Ch.2, pp.1-47.

IBID. 1, Ch.3, pp.1-3.

Van Oortmerssen, G., 1972
9 Symp. on Naval Hydrodynamics, Paris.

Van Oortmerssen, G., 1976
"The motions of a ship in shallow water", Ocean Eng., 3, pp.221-255.

Vlugts, J.H., August 1971
"The role of model tests and their correlation with full scale observations", Symp. on Offshore Hydrodynamics, N.S.M.B. publ. No.375, Wageningen.

Walpole, R.E. and Myers, R.H., 1978
"Probability and statistics for engineers and scientists", Second Ed., Collier MacMillan, London.

Wendel, K., 1950
"Hydrodynamische Massen und Hydrodynamische Massentragheitsmoment", Jb. Schiffbautech, Ges 44.

White, J.W.,1972
"Simply supported beams : Cantilevers", Crosby Lockwood, Great Britain.

Wilson, B.W., October 1958
"The energy problem in the mooring of ships exposed to waves", Princeton Univ. Conf. on berthing and handling in exposed locations.

Yeung, R.W., August 1973
"A singularity distribution method for free-surface flow problems with an oscillating body", Univ. of California, College of Engng, Report NA 73-6

Yeung, R.W., May 1981
"Added mass and damping of a vertical cylinder in finite-depth waters", Applied Ocean Res., Vol.3, No.3, pp.119-133.

Zwamborn, J.A. and van Wyk, A.C., May 1979
"Monitoring of ship motions in the Richards Bay harbour entrance channel", Proc. 25th Congr. PIANC, Edinburgh.

APPENDIX A

MODEL DIMENSIONS

AND LINES PLAN

APPENDIX A

MODEL DIMENSIONS

Model Scale		=	1:144
Length between perpendiculars	L_{pp}	=	2,Ø m
Breadth Amidships	B	=	285,7 mm
Draught Amidships	T	=	114,3 mm
Displacement	V	=	45,718 dm^3
Block Coefficient	C_b	=	Ø,7
Water plane area		=	Ø,4365 m^2
Moment of water plane area		=	Ø,Ø745 m^4
Metacentric Height	GM	=	1,593 m

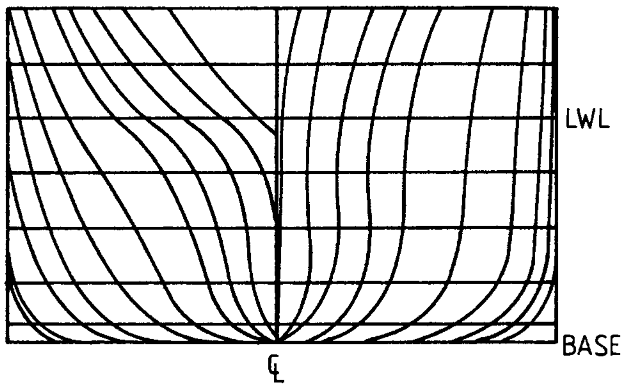
(assuming the distance from the centre of gravity to centre of buoyancy = 3Ø% of the draught.)

Number of station points	=	21
Distance between station points	=	Ø,1Ø m

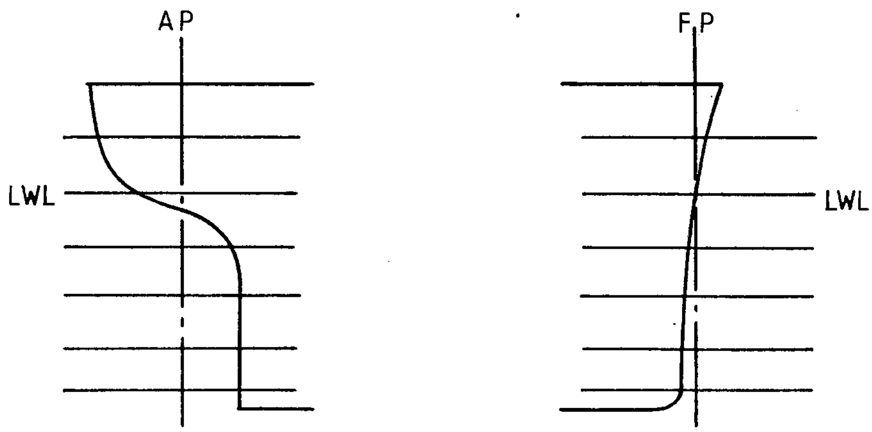
STAT. No.	DIST. FROM C. of G. (m)	HALF BEAM (m)	MEAN DRAFT (m)	AREA $\times 10^{-3}$ (m^2)	WETTED PERIMETER (m)
0	-1,050	0,0	0,0656	0,614	0,0330
1	-0,950	0,0286	0,0702	6,749	0,2625
2	-0,850	0,0594	0,0928	15,35	0,2980
3	-0,750	0,0889	0,0982	22,72	0,3345
4	-0,650	0,1120	0,1079	27,94	0,3765
5	-0,550	0,1271	0,1117	29,16	0,4070
6	-0,450	0,1371	0,1134	29,47	0,4350
7	-0,350	0,1413	0,1143	29,47	0,4625
8	-0,250	0,1427	0,1143	29,47	0,4800
9	-0,150	0,1429	0,1143	29,47	0,4825
10	-0,050	0,1429	0,1143	29,47	0,4850
11	0,050	0,1429	0,1143	29,47	0,4860
12	0,150	0,1429	0,1143	29,47	0,4845
13	0,250	0,1421	0,1143	29,47	0,4800
14	0,350	0,1397	0,1143	29,47	0,4745
15	0,450	0,1340	0,1135	29,16	0,4540
16	0,550	0,1230	0,1122	27,63	0,4320
17	0,650	0,1039	0,1101	25,17	0,3795
18	0,750	0,0770	0,1073	19,96	0,3500
19	0,850	0,0441	0,1015	9,820	0,2570
20	0,950	0,0116	0,0	0,0	0,0

TABLE A-1

MODEL SECTIONAL PROPERTIES

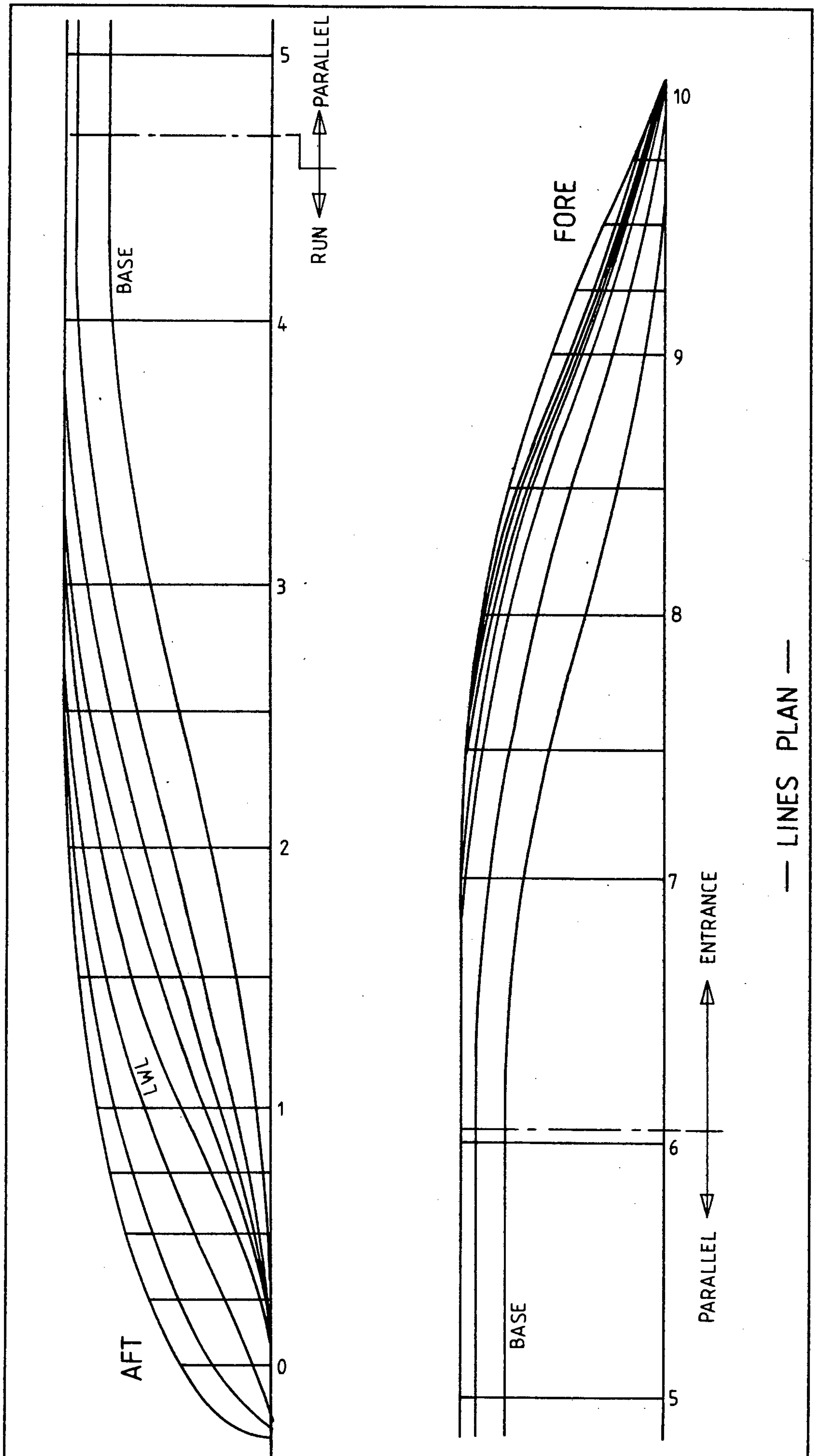


BODY PLAN



STERN PROFILE

STEM PROFILE


AFT
FORE
BASE
LWL
RUN
PARALLEL
ENTRANCE
0
1
2
3
4
5
6
7
8
9
10
— LINES PLAN —

APPENDIX B

COMPUTER LISTING OF

PROGRAM "SMISW"

```
1000 '**************************************************************
1010 '*                                                            *
1020 '*                  UNIVERSITY  OF  CAPE  TOWN                *
1030 '*                                                            *
1040 '*              DEPARTMENT  OF  CIVIL  ENGINEERING            *
1050 '*                                                            *
1060 '*                                                            *
1070 '*                      PROGRAM  -  "SMISW"                   *
1080 '*                                                            *
1090 '*        SHIP  MOTIONS  IN SHALLOW  SEAS  SUBLECT  TO        *
1100 '*                                                            *
1110 '*                       HEADING  WAVES                       *
1120 '*                                                            *
1130 '*       K R K  BLAKE                    SEPTEMBER  1986      *
1140 '*                                                            *
1150 '*                                                            *
1160 '*  This program calculates the predicted vertical motions    *
1170 '*  of a ship travelling in shallow water subject to heading  *
1180 '*  waves, given the hydrodynamic coefficients of the ship.   *
1190 '*                                                            *
1200 '*                                                            *
1210 '*                                                            *
1220 '**************************************************************
1230 '
1240 '**************************************************************
1250 '*                                                            *
1260 '*      DIMENSION ARRAYS AND READ IN DATA FROM FILES          *
1270 '*                                                            *
1280 '**************************************************************
1290 '
1300 '
1310 CLS
1320 '
1330 DIM A(2,2) , B(2,2) , C(2,2) , I(2,2) , R(2,2) , GRP(5,100)
1340 '
1350 INPUT"Type in the name of the ship data file"; NME$
1360 PRINT
1370 INPUT"Type in the name of the run data file"; MME$
1380 '
1390 OPEN "I",#1,NME$
1400 OPEN "I",#2,MME$
1410 '
1420 DEF FNCOSH(Z) = .5 * ( EXP(Z) + EXP(-Z))
1430 DEF FNSINH(Z) = .5 * ( EXP(Z) - EXP(-Z))
1440 G = 9.810001
1450 PI = 3.141592654#
1460 '
1470 INPUT#1, N, SPACEX, BEAM, DRAFT, SHPLEN, R
1480 '
1490 DIM  MDRAFT(N), S(N), X(N), YW(N), WETPTR(N)
1500 '
1510 FOR I = 1 TO N
1520  INPUT#1, X(I), YW(I), MDRAFT(I), S(I), WETPTR(I)
1530 NEXT I
1540 '
```

```
155Ø INPUT#2, H, U, WFRQ1, WFRQ2, STP
156Ø '
157Ø CLOSE #1
158Ø CLOSE #2
159Ø '
16ØØ    J = Ø
161Ø '
162Ø ADMS = 86.446
163Ø HDPS = 195.81
164Ø '
165Ø  ADM = ADMS / .Ø228696
166Ø  HDP = HDPS / .3927
167Ø '
168Ø FOR QZ = WFRQ1 TO WFRQ2 STEP STP
169Ø '
17ØØ J = J + 1
171Ø '
172Ø THDAMP = Ø
173Ø TADMAS = Ø
174Ø HDPMT2 = Ø
175Ø ADMMT2 = Ø
176Ø SARMT2 = Ø
177Ø MCONST = Ø
178Ø FCONST = Ø
179Ø MCON1  = Ø
18ØØ MCON2  = Ø
181Ø MCON3  = Ø
182Ø MCON4  = Ø
183Ø MCON5  = Ø
184Ø MCON6  = Ø
185Ø FCON1  = Ø
186Ø FCON2  = Ø
187Ø FCON3  = Ø
188Ø FCON4  = Ø
189Ø FCON5  = Ø
19ØØ FCON6  = Ø
191Ø  DISP  = Ø
192Ø   TME  = Ø
193Ø '
194Ø '
195Ø '***************************************************************
196Ø '*                                                             *
197Ø '*          CALCULATION OF REQUIRED PARAMETERS                 *
198Ø '*                                                             *
199Ø '***************************************************************
2ØØØ '
2Ø1Ø '
2Ø2Ø   R2G = 2 * R * G
2Ø3Ø  WAMP = 1!
2Ø4Ø WFREQ = WFRQ1 + ( J - 1 ) * STP
2Ø5Ø '
2Ø6Ø GOSUB 413Ø
2Ø7Ø '
2Ø8Ø  WNUM = TEST
2Ø9Ø '
```

```
2100  WE    = WFREQ + WNUM * U
2110 '
2120  PERD = 2 * PI / WFREQ
2130 '
2140  GOSUB 4350
2150 '
2160  WLEN = TEST * PERD
2170 '
2180  DWNUM = WFREQ * WFREQ / G
2190 '
2200 '
2210     DELTAX = SPACEX / 2
2220 '
2230  FOR COUNT = 1 TO N
2240 '
2250   IF COUNT = N THEN DELTAX = SPACEX / 2
2260   X1 = X(COUNT) * DELTAX
2270   X2 = X(COUNT) * X1
2280 '
2290   ADMASS = ADM * S(COUNT)
2300   HDAMP  = HDP * WETPTR(COUNT)
2310 '
2320   THDAMP = THDAMP + HDAMP     * DELTAX
2330   TADMAS = TADMAS + ADMASS    * DELTAX
2340   DISP   = DISP   + S(COUNT)  * DELTAX
2350   ADMMT2 = ADMMT2 + ADMASS    * X2
2360   SARMT2 = SARMT2 + S(COUNT)  * X2
2370   HDPMT2 = HDPMT2 + HDAMP     * X2
2380 '
2390 '
2400 '**************************************************************
2410 '*                                                            *
2420 '*             CALCULATION OF EXCITING FORCES                 *
2430 '*                                                            *
2440 '**************************************************************
2450 '
2460 '
2470 FCONST = FNCOSH( WNUM * ( H - MDRAFT(COUNT))) / ( FNCOSH( WNUM

                 * H )) * COS( WNUM * X(COUNT) )
2480 MCONST = FNCOSH( WNUM * ( H - MDRAFT(COUNT))) / ( FNCOSH( WNUM

                 * H )) * SIN( WNUM * X(COUNT) )
2490 '
2500 FCON1   = FCON1   + FCONST * YW(COUNT) * DELTAX
2510 FCON2   = FCON2   + FCONST * ADMASS    * DELTAX
2520 FCON3   = FCON3   + MCONST * HDAMP     * DELTAX
2530 FCON4   = FCON4   + MCONST * YW(COUNT) * DELTAX
2540 FCON5   = FCON5   + MCONST * ADMASS    * DELTAX
2550 FCON6   = FCON6   + FCONST * HDAMP     * DELTAX
2560 '
2570 MCON1   = MCON1   + FCONST * YW(COUNT) * X1
2580 MCON2   = MCON2   + FCONST * ADMASS    * X1
2590 MCON3   = MCON3   + MCONST * HDAMP     * X1
2600 MCON4   = MCON4   + MCONST * YW(COUNT) * X1
```

```
2610 MCON5   = MCON5   + MCONST * ADMASS     * X1
2620 MCON6   = MCON6   + FCONST * HDAMP      * X1
2630 '
2640  DELTAX = SPACEX
2650  NEXT COUNT
2660 '
2670 IYY = R * SARMT2
2680 '
2690  FA1 = R2G * WAMP * FCON1 - WFREQ * WAMP * WE * FCON2 - WFREQ

            * WAMP * FCON3
2700  FA2 = R2G * WAMP * FCON4 - WFREQ * WAMP * WE * FCON5 + WFREQ

            * WAMP * FCON6
2710  MA1 = - R2G * WAMP * MCON1 + WFREQ * WAMP * MCON3 + WFREQ * U

            * WAMP * FCON5 + WFREQ * WE * WAMP * MCON2
2720  MA2 = - R2G * WAMP * MCON4 - WFREQ * WAMP * MCON6 - WFREQ * U

            * WAMP * FCON2 + WFREQ * WE * WAMP * MCON5
2730 '
2740  RFA = FA1
2750  IFA = FA2
2760  RMA = MA1
2770  IMA = MA2
2780 '
2790 '
2800 '**************************************************************
2810 '*                                                            *
2820 '*             INPUT OF HYDRODYNAMIC COEFFICIENTS             *
2830 '*                                                            *
2840 '**************************************************************
2850 '
2860 '
2870 A(1,1) = ****
2880 A(1,2) = ****
2890 A(2,1) = A(1,2)
2900 A(2,2) = ****
2910 '
2920 B(1,1) = ****
2930 B(1,2) = ****
2940 B(2,1) = B(1,2)
2950 B(2,2) = ****
2960 '
2970 C(1,1) = ****
2980 C(1,2) = ****
2990 C(2,1) = C(1,2)
3000 C(2,2) = ****
3010 '
3020 '
3030 '**************************************************************
3040 '*                                                            *
3050 '*               SET COUPLING COEFFICIENT SIGNS               *
3060 '*                                                            *
3070 '**************************************************************
3080 '
```

```
3090 '
3100 A(1,2) = A(1,2) * -1
3110 A(2,1) = A(2,1) * -1
3120 B(1,2) = B(1,2) * -1
3130 B(2,1) = B(2,1) * -1
3140 C(1,2) = C(1,2) * -1
3150 C(2,1) = C(2,1) * -1
3160 '
3170 '
3180 '*************************************************************
3190 '*                                                           *
3200 '*                 SET UP COMPLEX MATRICES                   *
3210 '*                                                           *
3220 '*************************************************************
3230 '
3240 '
3250 FOR I = 1 TO 2
3260   FOR IJ = 1 TO 2
3270      R(I,IJ) = - WFREQ * WFREQ * A(I,IJ) + C(I,IJ)
3280      I(I,IJ) = - WFREQ * B(I,IJ)
3290   NEXT IJ
3300 NEXT I
3310 '
3320 '
3330 '*************************************************************
3340 '*                                                           *
3350 '*            DIRECT SOLUTION OF COMPLEX MATRIX              *
3360 '*                                                           *
3370 '*************************************************************
3380 '
3390 '
3400 K1 = R(1,1) * R(2,2) - I(1,1) * I(2,2) - R(2,1) * R(1,2) +

          I(1,2) * I(2,1)
3410 K2 = R(1,1) * I(2,2) + I(1,1) * R(2,2) - I(2,1) * R(1,2) -

          I(1,2) * R(2,1)
3420 K3 = K1 * K1 + K2 * K2
3430 '
3440 REHV = ( RFA * ( K1 * R(2,2) + K2 * I(2,2) ) - IFA * ( K1 *

            I(2,2) - K2 *  R(2,2) ) - RMA * ( K1 * R(1,2) + K2 *

            I(1,2) ) + IMA * ( K1 * I(1,2) - K2 * R(1,2)  ) ) / K3
3450 IMHV = ( IFA * ( K1 * R(2,2) + K2 * I(2,2) ) + RFA * ( K1 *

            I(2,2) - K2 *  R(2,2) ) - IMA * ( K1 * R(1,2) + K2 *

             I(1,2) ) - RMA * ( K1 * I(1,2) - K2 * R(1,2)  ) ) / K3
3460 REPT = ( -RFA * ( K1 * R(2,1) + K2 * I(2,1) ) + IFA * ( K1 *

            I(2,1) - K2 * R(2,1) ) + RMA * ( K1 * R(1,1) + K2 *

            I(1,1) ) - IMA * ( K1 * I(1,1) - K2 * R(1,1)  ) ) / K3
```

```
3470 IMPT = ( -IFA * ( K1 * R(2,1) + K2 * I(2,1) ) - RFA * ( K1 *
          I(2,1) - K2 * R(2,1) ) + IMA * ( K1 * R(1,1) + K2 *
          I(1,1) ) + RMA * ( K1 * I(1,1) - K2 * R(1,1)  ) ) / K3
3480 '
3490  HAMP = ( REHV * REHV + IMHV * IMHV )^.5
3500  PAMP = ( REPT * REPT + IMPT * IMPT )^.5
3510 PPANG =  ATN( IMPT / REPT )
3520 HPANG =  ATN( IMHV / REHV )
3530 '
3540 IF REPT < 0 THEN PPANG = PPANG + PI : GOTO 3560
3550 IF IMPT < 0 THEN PPANG = PPANG + 2 * PI
3560 IF REHV < 0 THEN HPANG = HPANG + PI : GOTO 3580
3570 IF IMHV < 0 THEN HPANG = HPANG + 2 * PI
3580 '
3590 '
3600 '*************************************************************
3610 '*                                                           *
3620 '*              STORE VALUES IN MATRIX FORM                  *
3630 '*                                                           *
3640 '*************************************************************
3650 '
3660 '
3670 GRP(1,J) = WLEN / SHPLEN
3680 GRP(2,J) = HAMP / WAMP
3690 GRP(3,J) = PAMP / (WAMP * WNUM )
3700 GRP(4,J) = HPANG
3710 GRP(5,J) = PPANG
3720 '
3730 '
3740 NEXT QZ
3750 '
3760 '
3770 '*************************************************************
3780 '*                                                           *
3790 '*                     PRINT OUT DATA                        *
3800 '*                                                           *
3810 '*************************************************************
3820 '
3830 '
3840 LPRINT "WAVE LENGTH    HEAVE AMP.      PITCH AMP.      HEAVE";
3850 LPRINT " PHASE      PITCH PHASE"
3860 LPRINT "-----------    ----------      ----------              AN";
3870 LPRINT "GLE             ANGLE"
3880 LPRINT "SHIP LENGTH    WAVE AMP.    WAVE AMP.* WAVE NUM "
3890 LPRINT
3900 '
3910 FOR IJ = 1 TO J
3920    FOR I = 1 TO 5
3930        LPRINT TAB((I-1)*16) USING "##.###^^^^";GRP(I,IJ);
3940    NEXT I
3950 NEXT IJ
3960 '
3970 INPUT "Enter name of data file for storage  ",D$
```

```
398Ø OPEN "O",#1,D$
399Ø PRINT #1, J
4ØØØ '
4Ø1Ø FOR I = 1 TO J
4Ø2Ø     PRINT #1,  GRP(1,I), GRP(2,I), GRP(3,I)
4Ø3Ø NEXT I
4Ø4Ø '
4Ø5Ø CLOSE#1
4Ø6Ø '
4Ø7Ø '
4Ø8Ø END
4Ø9Ø '
41ØØ '
411Ø '**************************************************************
412Ø '*                                                            *
413Ø '*       CALCULATION OF SHALLOW WATER WAVE NUMBER             *
414Ø '*                                                            *
415Ø '**************************************************************
416Ø '
417Ø '
418Ø MAX = 1ØØ
419Ø MIN = Ø
42ØØ WCON = WFREQ * WFREQ / G
421Ø TEST = ( MAX + MIN ) / 2
422Ø Z1 = EXP( TEST * H )
423Ø Z2 = EXP( -TEST * H )
424Ø RESULT = TEST * ( Z1 - Z2 ) / ( Z1 + Z2 )
425Ø IF ABS( WCON - RESULT ) <= .ØØ1 THEN  RETURN
426Ø IF RESULT > WCON THEN MAX = TEST ELSE MIN = TEST
427Ø GOTO 421Ø
428Ø '
429Ø '**************************************************************
43ØØ '*                                                            *
431Ø '*        CALCULATION OF WAVELENGTH BY AIRY THEORY            *
432Ø '*                                                            *
433Ø '**************************************************************
434Ø '
435Ø MAX = 1ØØ
436Ø MIN = .1
437Ø WCON = TEST * TEST
438Ø TEST = ( MAX + MIN ) / 2
439Ø Z1 = EXP( 2 * PI * H / ( TEST * PERD ) )
44ØØ Z2 = EXP( - 2 * PI * H / ( TEST * PERD ) )
441Ø RESULT = G * TEST * PERD / ( 2 * PI ) * ( Z1 - Z2 ) /

              ( Z1 + Z2 ) - TEST^2
442Ø IF ABS( RESULT ) <= .ØØ1 THEN  RETURN
443Ø IF RESULT > Ø THEN MIN = TEST ELSE MAX = TEST
444Ø GOTO 438Ø
445Ø '
446Ø '
447Ø '**************************************************************
448Ø '
449Ø END
45ØØ '
451Ø '**************************************************************
```

APPENDIX C

SOLUTION OF

COMPLEX MATRICES

APPENDIX C

SOLUTION OF COMPLEX MATRICES

Given a matrix in the form

$$\begin{array}{ccc} C(1,1)\ ,\ C(1,2) & . & Z(1) \\ C(2,1)\ ,\ C(2,2) & . & Z(2) \end{array} \quad = \quad \begin{array}{c} F(1) \\ F(2) \end{array}$$

where $C(j,k) = R(j,k) + i.I(j,k)$

and $F(j) = G(j) + i.H(j)$

Then the solution must be in the form

$$Z(j) = X(j) + i.Y(j)$$

This solution can be shown to be given as:-

$$X(1) = (G(1) * (K1 * R(2,2) + K2 * I(2,2)) - H(1) * (K1 * I(2,2) - K2 * R(2,2) - G(2) * (K1 * R(1,2) + K2 * I(1,2)) + H(2) * (K1 * I(1,2) - K2 * (1,2))) / K3$$

$$Y(1) = (H(1) * (K1 * R(2,2) + K2 * I(2,2)) + G(1) * (K1 * I(2,2) - K2 * R(2,2) - H(2) * (K1 * R(1,2) + K2 * I(1,2)) - G(2) * (K1 * I(1,2) - K2 * R(1,2))) / K3$$

$$X(2) = (- G(1) * (K1 * R(2,1) + K2 * I(2,1)) + H(1) * (K1 * I(2,1) - K2 * R(2,1) + G(2) * (K1 * R(1,1) + K2 * I(1,1)) - H(2) * (K1 * I(1,1) - K2 * R(1,1))) / K3$$

$$Y(2) = (- H(1) * (K1 * R(2,1) + K2 * I(2,1)) - G(1) * (K1 * I(2,1) - K2 * R(2,1) + H(2) * (K1 * R(1,1) + K2 * I(1,1)) + G(2) * (K1 * I(1,1) - K2 * R(1,1))) / K3$$

where

$$K1 = R(1,1) * R(2,2) - I(1,1) * I(2,2) - R(2,1) * R(1,2) + I(1,2) * I(2,1)$$

$$K2 = R(1,1) * I(2,2) + I(1,1) * R(2,2) - I(2,1) * R(1,2) - I(1,2) * R(2,1)$$

$$K3 = K1^2 + K2^2$$

APPENDIX D

SPRING CALIBRATIONS

SPRING No. 1 TYPE: Tension

PROPERTIES: Number of coils = 24

Radius of coils = 19,25 mm

Diameter of wire = 3,65 mm

LOAD (N)	DEFLECTION (mm) Loading		Unloading	
60	479,5		482,0	
		(3,0)		(3,9)
70	482,5		485,9	
		5,7		5,5
80	488,2		491,4	
		5,9		6,1
90	494,1		497,5	
		6,1		5,6
100	500,2		503,1	
		6,3		5,7
110	506,5		508,8	
		6,7		5,6
120	513,2		514,4	
		6,0		4,8
130	519,2		519,2	

$x_1 = 6,12$ $x_2 = 5,55$

Mean $= (x_1 + x_2) / 2 = 5,84$ mm

$$\text{Stiffness} = \frac{10}{5,84\times10^{-3}}$$

$$= \underline{1713,80 \text{ N.m}^{-1}}$$

SPRING No. 2 TYPE: Tension

PROPERTIES: Number of coils = 24

Radius of coils = 19,25 mm

Diameter of wire = 3,65 mm

LOAD (N)	DEFLECTION (mm) Loading		Unloading	
60	480,8		482,1	
		(2,2)		(2,5)
70	483,0		484,6	
		5,0		5,4
80	488,0		490,0	
		5,5		5,9
90	493,5		495,9	
		6,0		6,6
100	499,5		502,7	
		6,1		4,3
110	505,6		507,0	
		6,0		5,5
120	511,6		512,5	
		6,1		5,2
130	517,7		517,7	

$x_1 = 5,78$ $x_2 = 5,48$

Mean $= (x_1 + x_2) / 2 = 5,63$ mm

$$\text{Stiffness} = \frac{10}{5,63 \times 10^{-3}}$$

$$= \underline{1776,20\ \text{N.m}^{-1}}$$

SPRING No. 3

TYPE: Linear Displacement Transducer

LOAD (N)	DEFLECTION (mm) Loading		DEFLECTION (mm) Unloading	
1	30,5		30,5	
		33,0		34,0
2	63,5		64,5	
		29,5		28,5
3	93,0		93,0	
1	31,0		27,0	
		33,5		37,0
2	64,5		64,0	
		29,5		30,0
3	94,0		94,0	
1	30,0		30,0	
		35,0		35,0
2	65,0		65,0	
		32,5		32,5
3	97,5		97,5	
		$x_1 = 32,17$		$x_2 = 32,83$

Mean $= (x_1 + x_2) / 2 = 32,50$ mm

$$\text{Stiffness} = \frac{1}{32,50\times10^{-3}}$$

$$= \underline{30,77\ \text{N.m}^{-1}}$$

SPRING No. 4

TYPE: Linear Displacement Transducer

LOAD (N)	DEFLECTION (mm) Loading		Unloading	
1	26,Ø		24,5	
		37,5		38,Ø
2	63,5		62,5	
		31,5		32,5
3	95,Ø		95,Ø	
1	24,Ø		24,5	
		38,Ø		38,Ø
2	62,Ø		62,5	
		34,Ø		33,5
3	96,Ø		96,Ø	
1	24,Ø		24,5	
		37,5		36,5
2	61,5		61,Ø	
		34,5		35,5
3	96,Ø		96,Ø	

x_1 = 35,5Ø $\qquad$ x_2 = 35,58

Mean $= (x_1 + x_2) / 2 = 35,54$

$$\text{Stiffness} = \frac{1}{35,54\text{x}1Ø^{-3}}$$

$$= \underline{28,14\ \text{N.m}^{-1}}$$

SPRING No. 5

TYPE: Linear Displacement Transducer (with dial guage)

<u>DEFLECTION (mm) PER Ø,49Ø5 N LOAD</u>

6,87
6,99
8,76
5,91
7,29
9,59
1Ø,38

Average = 7,97Ø mm

$$\text{Stiffness} = \frac{0{,}4905}{7{,}970\times10^{-3}}$$

$$= \underline{61{,}54\ \text{N.m}^{-1}}$$

SPRING No. 6

TYPE: Water spring in pitch mode

LOAD (N.m)	DEFLECTION (rad x1Ø$^{-3}$)			
	Loading		Unloading	
Ø	Ø		Ø,3Ø9	
		3,246		3,Ø91
2,925	3,246		3,4ØØ	
		3,Ø91		3,246
5,85Ø	6,337		6,646	
		3,245		2,936
8,775	9,582		9,582	
		2,937		2,937
11,7Ø	12,519		12,519	
		2,781		3,Ø9Ø
14,63	15,3ØØ		15,6Ø9	
		3,245		2,782
17,55	18,545		18,391	
		2,935		3,Ø89
2Ø,48	21,48Ø		21,48Ø	
		2,781		2,936
23,4Ø	24,261		24,416	
		2,471		2,471
26,33	26,732		26,887	
		3,244		3,Ø89
29,25	29,976		29,976	
		2,779		2,779
32,18	32,755		32,755	

x_1 = 2,9777 x_2 = 2,9496

Mean = (x_1 + x_2) / 2 = 2,9637 x1Ø$^{-3}$ rad

$$\text{Stiffness} = \frac{2{,}925}{2{,}9637\text{x}1\emptyset^{-3}} = \underline{986{,}9\ \text{N.m.rad}^{-1}}$$

Or if measured at the spring supports; Ø,647 m either side of the centre of gravity :-

$$\text{Stiffness} = \frac{2{,}925\ /\ \emptyset{,}647}{\emptyset{,}647.\tan(2{,}9637\text{x}1\emptyset^{-3})} = \underline{2357{,}1\ \text{N.m}^{-1}}$$

SPRING No. 7

TYPE: Water spring in heave mode

LOAD (N)	DEFLECTION (mm) Loading		Unloading	
0	751,3		751,3	
		1,3		1,3
5	750,0		750,0	
		1,1		1,1
10	748,9		748,9	
		1,1		1,0
15	747,8		747,9	
		1,1		1,4
20	746,7		746,5	
		1,2		1,0
25	745,5		745,5	
		1,0		1,0
30	744,5		744,5	
		1,0		1,0
35	743,5		743,5	
		1,1		1,3
40	742,4		742,2	
		1,1		1,0
45	741,3		741,2	
		1,1		1,0
50	740,2		740,2	
		1,2		1,2
55	739,0		739,0	
		1,2		1,2
60	737,8		737,8	

$x_1 = 1{,}13$ $\qquad$ $x_2 = 1{,}13$

Mean $= (x_1 + x_2) / 2 = 1{,}13$ mm

Stiffness $= \dfrac{5}{1{,}13\text{x}10^{-3}} = \underline{4444{,}4\ \text{N.m}^{-1}}$

APPENDIX E

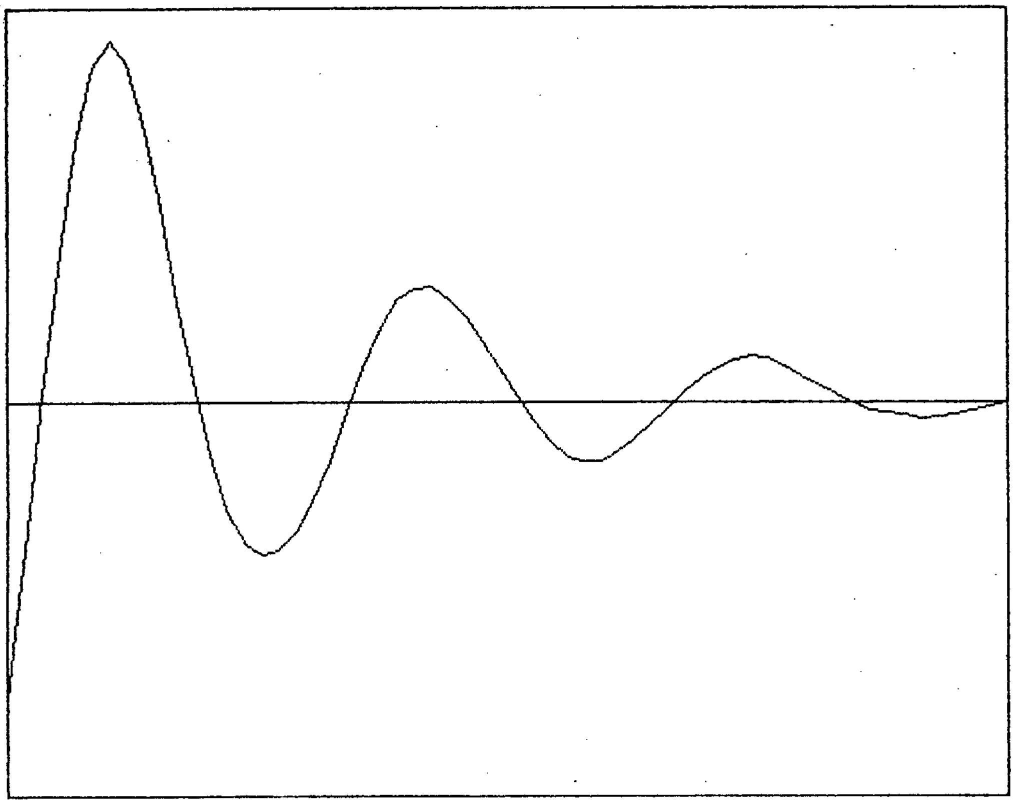
SAMPLE TEST RESULTS

TYPICAL HEAVE MOTION TEST RESULT

HEAVE DISPLACEMENT

-1.89	-0.91
0.06	0.95
1.66	2.13
2.30	2.16
1.78	1.24
0.65	0.08
-0.38	-0.70
-0.89	-0.96
-0.93	-0.81
-0.62	-0.36
-0.06	0.23
0.48	0.66
0.73	0.74
0.66	0.54
0.39	0.22
0.05	-0.11
-0.24	-0.34
-0.37	-0.36
-0.30	-0.22
-0.12	-0.03
0.07	0.16
0.23	0.28
0.30	0.29
0.24	0.17
0.12	0.06
0.01	-0.04
-0.06	-0.07
-0.09	-0.08
-0.07	-0.05
-0.02	0.00

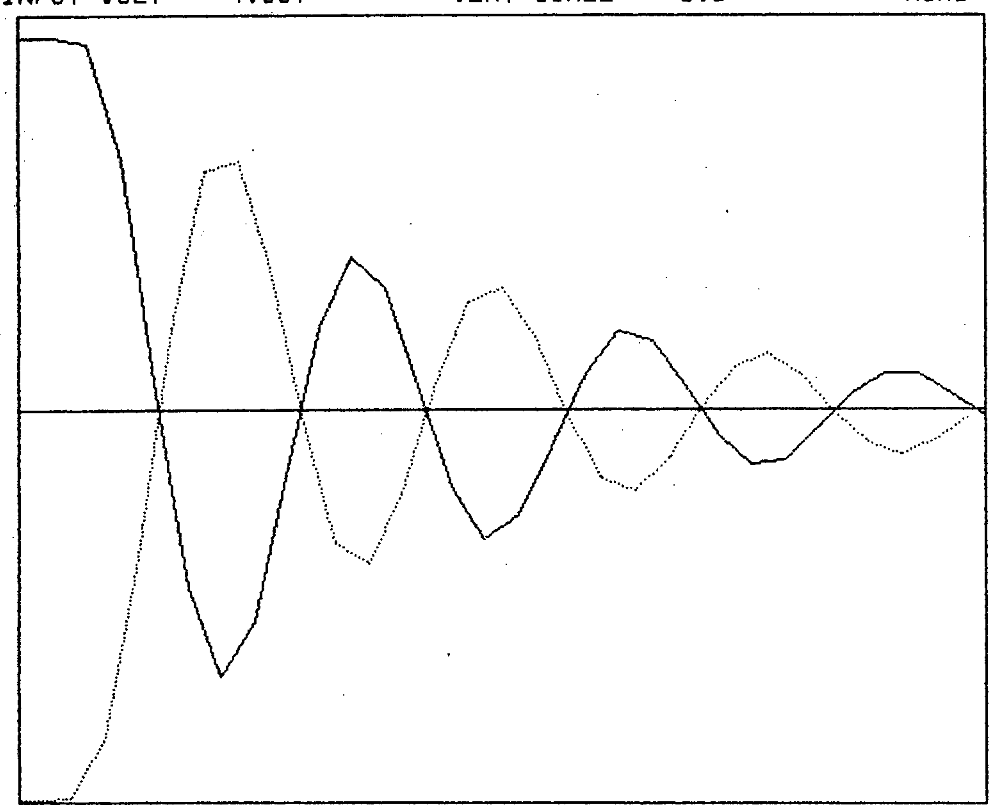
INPUT VOLT = 2.0001 VERT SCALE = 2.5 HORZ TIME = 3 sec



TYPICAL PITCH MOTION TEST RESULT

FORE	AFT
6.14	-6.46
6.14	-6.44
6.02	-5.41
4.13	-2.40
0.49	1.35
-2.93	3.94
-4.39	4.08
-3.50	2.36
-1.08	-0.22
1.38	-2.18
2.54	-2.54
2.03	-1.39
0.43	0.43
-1.24	1.78
-2.11	2.02
-1.73	1.21
-0.61	-0.10
0.63	-1.11
1.32	-1.33
1.14	-0.79
0.41	0.04
-0.43	0.71
-0.89	0.95
-0.81	0.59
-0.30	-0.00
0.27	-0.52
0.63	-0.71
0.61	-0.48
0.29	-0.12
-0.08	0.26

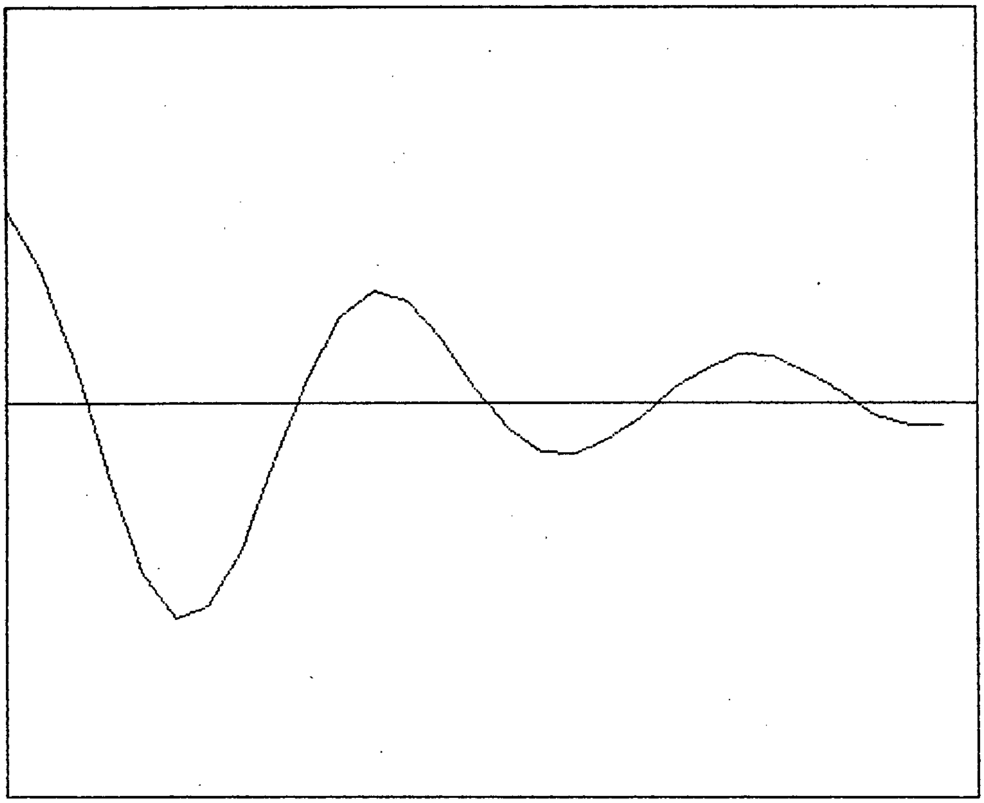
INPUT VOLT = 4.001 VERT SCALE = 6.5 HORZ TIME = 3 sec



TYPICAL COUPLED MOTION TEST RESULT

FORE	AFTR	DIFF
-4.76	-2.44	1.45
-3.01	-0.93	1.00
-0.85	0.53	0.33
1.26	1.47	-0.55
2.78	1.84	-1.27
3.44	1.68	-1.63
3.19	1.23	-1.54
2.34	0.69	-1.10
1.24	0.26	-0.46
0.20	0.02	0.20
-0.55	-0.08	0.65
-0.91	-0.04	0.86
-0.89	0.06	0.78
-0.55	0.16	0.48
-0.10	0.26	0.12
0.37	0.34	-0.20
0.69	0.40	-0.36
0.83	0.44	-0.37
0.79	0.44	-0.26
0.61	0.38	-0.11
0.35	0.32	0.11
0.06	0.22	0.28
-0.18	0.14	0.37
-0.30	0.08	0.36
-0.28	0.02	0.24
-0.16	0.02	0.08
0.04	0.04	-0.09
0.22	0.12	-0.16
0.33	0.18	-0.17
0.35	0.24	0.00

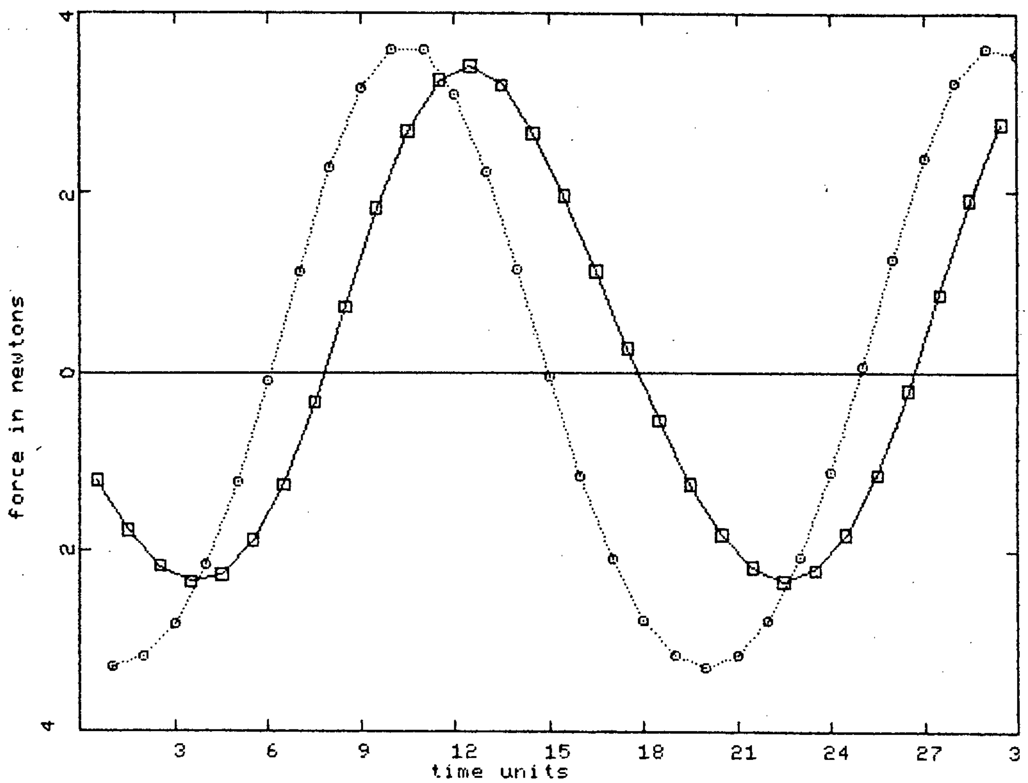
INPUT VOLT = 4.0002 VERT SCALE = 3



TYPICAL EXCITING FORCE TEST RESULT

FORE (N)	AFTR (N)	HEAVE (N)	PITCH (Nm)
-1.20	-3.30	-2.39	1.17
-1.77	-3.18	-2.58	0.78
-2.17	-2.80	-2.53	0.35
-2.35	-2.15	-2.23	-0.10
-2.26	-1.22	-1.65	-0.55
-1.88	-0.09	-0.83	-0.96
-1.26	1.14	0.17	-1.25
-0.33	2.28	1.24	-1.34
0.73	3.15	2.22	-1.21
1.82	3.60	2.93	-0.87
2.69	3.58	3.28	-0.39
3.25	3.09	3.21	0.16
3.42	2.23	2.77	0.70
3.20	1.15	2.04	1.16
2.67	-0.04	1.14	1.52
1.96	-1.15	0.19	1.75
1.13	-2.09	-0.69	1.80
0.27	-2.76	-1.45	1.70
-0.53	-3.15	-2.02	1.47
-1.24	-3.29	-2.41	1.14
-1.81	-3.15	-2.58	0.75
-2.18	-2.76	-2.51	0.33
-2.33	-2.06	-2.16	-0.14
-2.21	-1.11	-1.56	-0.59
-1.81	0.05	-0.71	-0.99
-1.15	1.26	0.29	-1.25
-0.21	2.38	1.36	-1.33
0.87	3.22	2.31	-1.18
1.92	3.62	2.98	-0.82
2.78	3.54	0.00	0.00

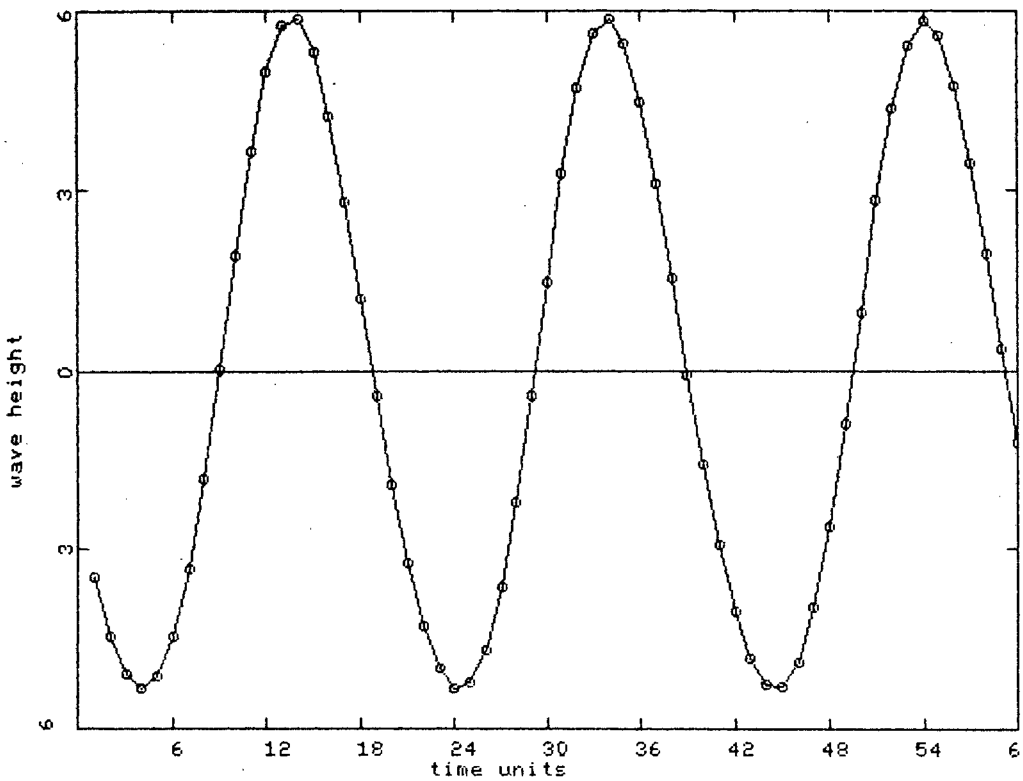
INPUT VOLT = 10.00065



TYPICAL WAVE FORM

DEPTH (mm)	DEPTH (mm)
-3.48	-4.45
-5.09	-5.33
-5.14	-4.47
-3.33	-1.81
0.01	1.92
3.67	5.00
5.76	5.86
5.33	4.23
2.79	1.20
-0.43	-1.92
-3.25	-4.27
-4.99	-5.32
-5.23	-4.67
-3.64	-2.20
-0.42	1.47
3.29	4.72
5.61	5.86
5.47	4.48
3.11	1.56
-0.06	-1.57
-2.93	-4.03
-4.83	-5.28
-5.31	-4.89
-3.97	-2.61
-0.89	0.99
2.85	4.39
5.42	5.83
5.60	4.75
3.46	1.98
0.35	-1.20

WAVE HEIGHT = 11.19 mm



APPENDIX F

EXAMINATIONS WRITTEN BY THE AUTHOR

APPENDIX F

Examinations written by the author to complete the requirements of the degree.

EXAMINATION			CREDITS
P.O. 1a	Physical Oceanography 1a	(July 1985)	5
P.O. 1b	Physical Oceanography 1b	(Nov. 1985)	5
CE 5B17	Finite Element Analysis	(July 1985)	4
CE 5D5	Advanced Hydraulic Structures	(Nov. 1985)	5
CIV 516F	Coastal Hydraulics	(July 1986)	5
Thesis			20
		Total	44

Total credit requirements of this degree 40

UNIVERSITY OF CAPE TOWN - JUNE EXAMINATION 1985

PHYSICAL OCEANOGRAPHY 1a

TIME: 3 HOURS TOTAL MARKS: 150

Answer ALL questions in SECTION A

Answer TWO questions EACH from SECTION B and SECTION C

Use separate books for Sections B and C.

Section A

Answer ALL questions in this section.

1. Define a barotropic current. (2)

2. Briefly explain the Coriolis force. (2)

3. Sketch a reversing thermometer and explain how it is used. (2)

4. Describe three different instruments used to determine temperature at different depths. (3)

5. Define the terms: potential temperature, salinity, sigma-t, specific volume anomaly, standard ocean. (5)

6. What is the solar constant? (1)

7. What is black body radiation? (1)

8. What are the units of pressure? geopotential? (2)

9. Give examples (with sketches) of fronts in the atmosphere and ocean. (2)

10. What is a Kelvin wave? (1)

11. What three factors are required in order for waves to be present. (3)

12. What is a wave spectrum? (2)

13. What do you understand by "dispersion of water waves" (1)

14. What is the diurnal inequality of the tide - when does it vanish? (1)

15. What are typical oxygen concentrations in the sea? (1)

16. What is the temperature and salinity of Antarctic Bottom Water? (1)

Section B

Answer TWO questions ONLY from this section.

(i) Describe briefly, with sketches where necessary, the surface wind systems around southern Africa. Distinguish between winter and summer circulation patterns and draw surface pressure maps typical of winter and summer. (30)

(ii) Describe briefly the major surface currents around southern Africa. Draw a map of the surface currents and indicate the major topographic features in relation to the currents. (30)

(iii) Given that there is no global gain or loss of heat in the ocean, write down the heat balance equation for the ocean. Describe the various terms and give the magnitude. Pay particular attention to the detail of how the solar radiation reaches the sea surface. (30)

(iv) Describe the circulation and water masses in a section from pole to pole through the Atlantic ocean. Describe the circulation in a vertical section from pole to pole through the atmosphere. What is the jet stream and where can it be found? (30)

Section C

Answer TWO questions ONLY from this section.

(a) Small amplitude gravity waves can be found as a solution to the relevant equations, provided that the dispersion relation holds:

$$\sigma^2 = gk \tanh (kd)$$

where $\sigma = \frac{2\pi}{T}$, T the period of the waves

$k = \frac{2\pi}{L}$, L the wavelength of the waves

$g = 10$ m/s, d is the depth of water.

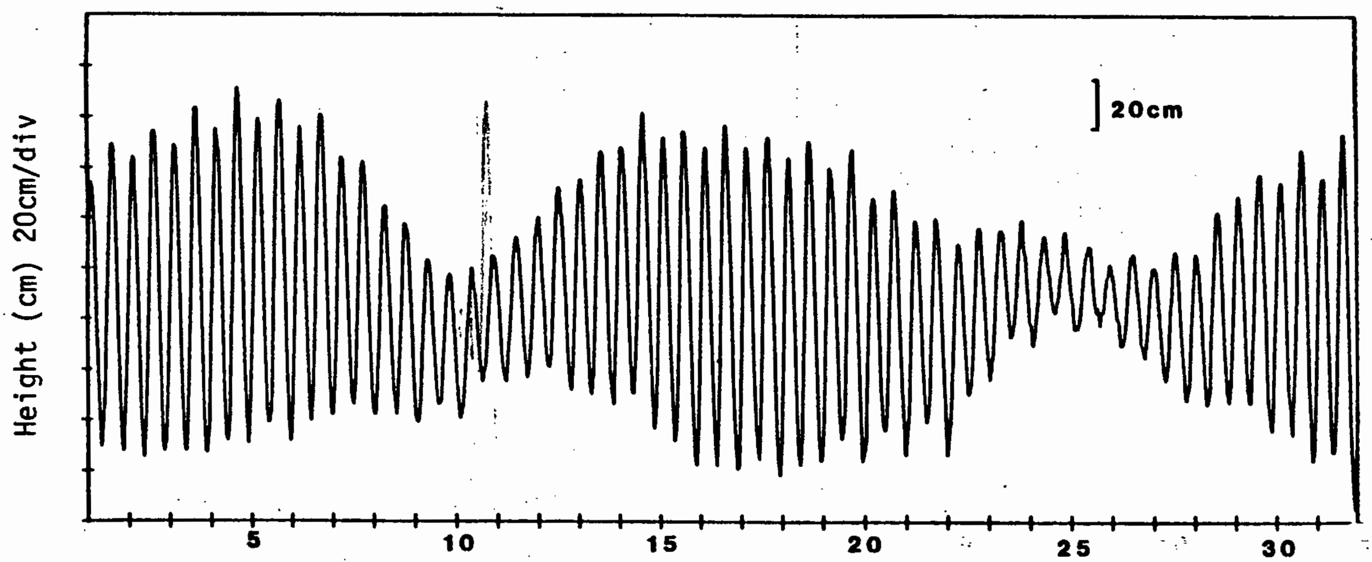
Define the terms for "deep" and "shallow" water waves and give the approximate relations for the speed of deep and shallow water waves. (12)

Given that the period T of the waves is independent of the water depth, discuss what happens when gravity waves approach the beach. (12)

If there is a long wave with a wavelength of 1000 km, what will its speed be in water of 4 km depth? (6)

(b) Assuming simple tidal theory, discuss how the tides are generated in principle by the sun and moon. Which tide producing force dominates?

Consider the following tidal record for Simons Bay carefully and discuss various features of the tide there. Expand your discussion to include details of tides around the south African coast. (30)



Simons bay sea level October 1982

(c) Starting with the equations of motion for fluids as a rotating earth, make appropriate assumptions so that you can derive the geostrophic velocity equation. (15)

Calculate the geostrophic, barotropic current at 30^oS latitude if the sea surface slopes up away from the coast by 1 metre in 100 km. If this current is on the east coast of Australia, in which direction does it flow? (15)

NOTES:

$$\frac{Du}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + T_x$$

$$\frac{Dv}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + T_y$$

$$\frac{Dw}{Dt} = g - \frac{1}{\rho}\frac{\partial p}{\partial z} + T_z$$

$f = 2\ \Omega \sin \phi$ rad/s

$\rho = 1000\ kg/m^3$, $g = 10 m/s^2$

(d) Discuss briefly the irregular phenomenon of El Nino in the Pacific Ocean giving some details of the Walker circulation and the southern oscillation. (30)

(e) Discuss the classical ideas of Ekman relating the vertical two dimensional current structure with depth to the wind stress at the surface.

By considering a lateral boundary and an equatorward wind stress, extend these ideas to give a simple model of coastal upwelling. (30)

(f) Write short notes on:
The Phillips-Miles theories of wind wave generation.
Wave spectra.
Sampling temperature and salinity in the sea.
The origin of the most dense water mass in the sea. (30)

UNIVERSITY OF CAPE TOWN

DEPARTMENT OF PHYSICAL OCEANOGRAPHY

Physical Oceanography 1b
November 1985

Time : 3 Hours
Answer ALL questions in Section A (40 marks)
and THREE whole questions from Section B

Total : 100 marks

SECTION A

Short questions. Answer ALL questions in this section.

1. Discuss the concept of a level surface and its relation to the local vertical, explaining the nature of the various contributing phenomena.

2. Explain the quasi-static approximation in the vertical balance of forces in ocean dynamics.

3. What quantity provides a contrast between the following curved currents; the South Atlantic gyre, inertial oscillations, and whirls in river flows.

4. Discuss the horizontal divergence of a geostrophic flow with particular emphasis on the relevance of the beta effect.

5. Explain what is meant by a dispersive wave. Is the Kelvin wave dispersive?

6. What is an amphidromic point?

7. Explain how wave energy can be concentrated at particular points on a coastline.

8. Explain how stratification can restrict the transfer of turbulence down the water column.

9. Discuss how the large scale wind field over the South Atlantic gyre can give rise to a deep central thermocline.

10. Explain how the Agulhas current can be "topographically steered" down the south east coast of South Africa.

(40 marks)

SECTION B

Long questions. Answer THREE whole questions only.

Question 1

The continuity equation can be interpreted as a balance between horizontal divergence and the vertical variation of vertical velocity.

Hence explain

(a) The deepening of a well-mixed layer
(b) The vertical velocity structure in a geostrophic flow
(c) The vertical velocity structure in a planetary scale geostrophic flow.

(20 marks)

Question 2

Many of the large scale surface currents of the ocean are confined to the upper layers of the ocean. Give a full explanation of the sense of the sea surface isobaric surface and the sense of the subsurface isopycnal surfaces associated with such a surface-confined geostrophic current in the Southern Hemisphere.

(20 marks)

Question 3

(a) For surface gravity waves on the ocean, the general geophysical fluid dynamic equations can be simplified considerably. By introducing two appropriate length scales and a time scale, show how the equations can be simplified to give:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x}$$

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p}{\partial z} - g$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

Simplify these equations further by splitting the pressure into dynamic and static contributions. Show that these two contributions to the pressure operate at vastly different scales. (You can consider typical waves to have amplitudes of 1 metre, wavelengths of 100 m, and periods of 10 seconds).

(b) Wave energy travels at the "group velocity $C_g = \frac{d\sigma}{dk}$. By considering special simplifications of the dispersion relation

$$\sigma^2 = gk \tanh(kh)$$

find the group velocity in "deep" and "shallow" water in terms of the phase velocity.

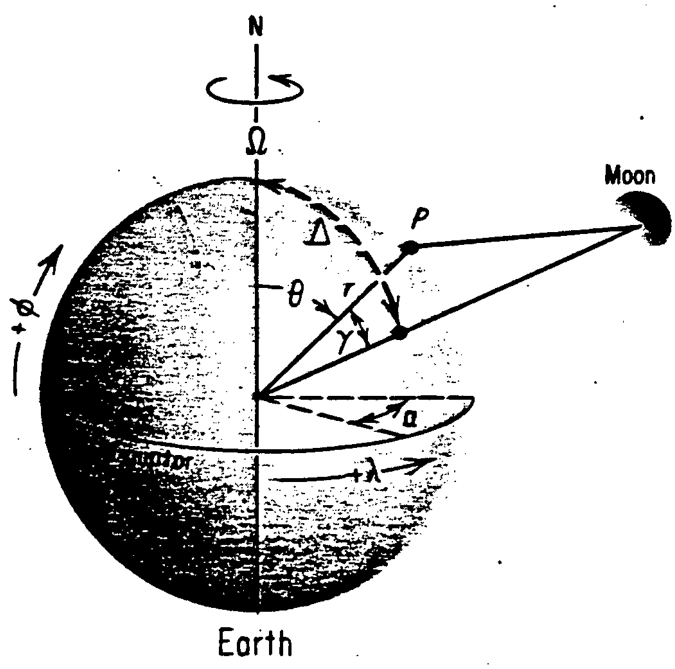
(20 marks)

Question 4

(a) In order to determine the tidal attraction due to the moon, we have to consider the variations in the angle γ which is subtended between the sublunar point M and a position P on the earth's surface. With the use of spherical trignometry, the term $(\cos^2\gamma - \frac{1}{3})$ in the tidal potential can be written as

$$\frac{3}{2}\left(\frac{1}{6}+\frac{1}{2}\cos 2\Delta\right)\left(\cos^2\theta-\frac{1}{3}\right)$$
$$+\ \frac{1}{2}\sin 2\Delta \cos\alpha \sin 2\theta$$
$$+\ \frac{1}{2}\left(\frac{1}{2}-\frac{1}{2}\cos 2\Delta\right)\cos 2\alpha \sin^2\theta$$

where θ is the co-latitude of P
Δ is the declination of moon
α is the transit angle (see accompanying diagram)



In the light of this equation, discuss the contribution to tides at long periods, diurnal periods and semidiurnal periods. Show also at which latitudes the long period, diurnal, and semidiurnal tides will be weak or absent.

(b) What will the speed of propagation of the tide be if the M_2 component has a wave length of 920 km and period of 12.42 hours? If the tide is considered as a shallow water wave, what would the the appropriate depth be to give the speed found above?

(20 marks)

Question 5

What is the cause of the surface layer phenomenon known as "ice-edge upwelling"? Provide the mechanism and give an opinion as to its frequency of occurrence in the Antarctic.

(20 marks)

Question 6

The full form of the conservation of potential vorticity of a wind forced surface layer of constant density is

$$\frac{d}{dt}\left(\frac{\zeta+f}{h}\right)=\frac{1}{h^2\rho_0}\left(\frac{\partial s^y}{\partial x}-\frac{\partial s^x}{\partial y}\right)$$

where ζ is the relative vorticity and h the thickness of the layer, f is the planetary vorticity and ρ_0 the density. The wind stress is (s^x, s^y) and the total time derivative follows the current. This conservation law is to be applied to an ocean gyre and western boundary current in the southern hemisphere.

Estimate the size of the right hand side and show that it is only important for time scales which are very long compared with $\frac{1}{f}$.

Explain how the fast western boundary current responds to its poleward flow along the boundary, and how the slow return gyre current supplies the "lost" vorticity.

(20 marks)

UNIVERSITY OF CAPE TOWN

DEPARTMENT OF CIVIL ENGINEERING

FINAL EXAM. MONDAY, 24 JUNE 1985

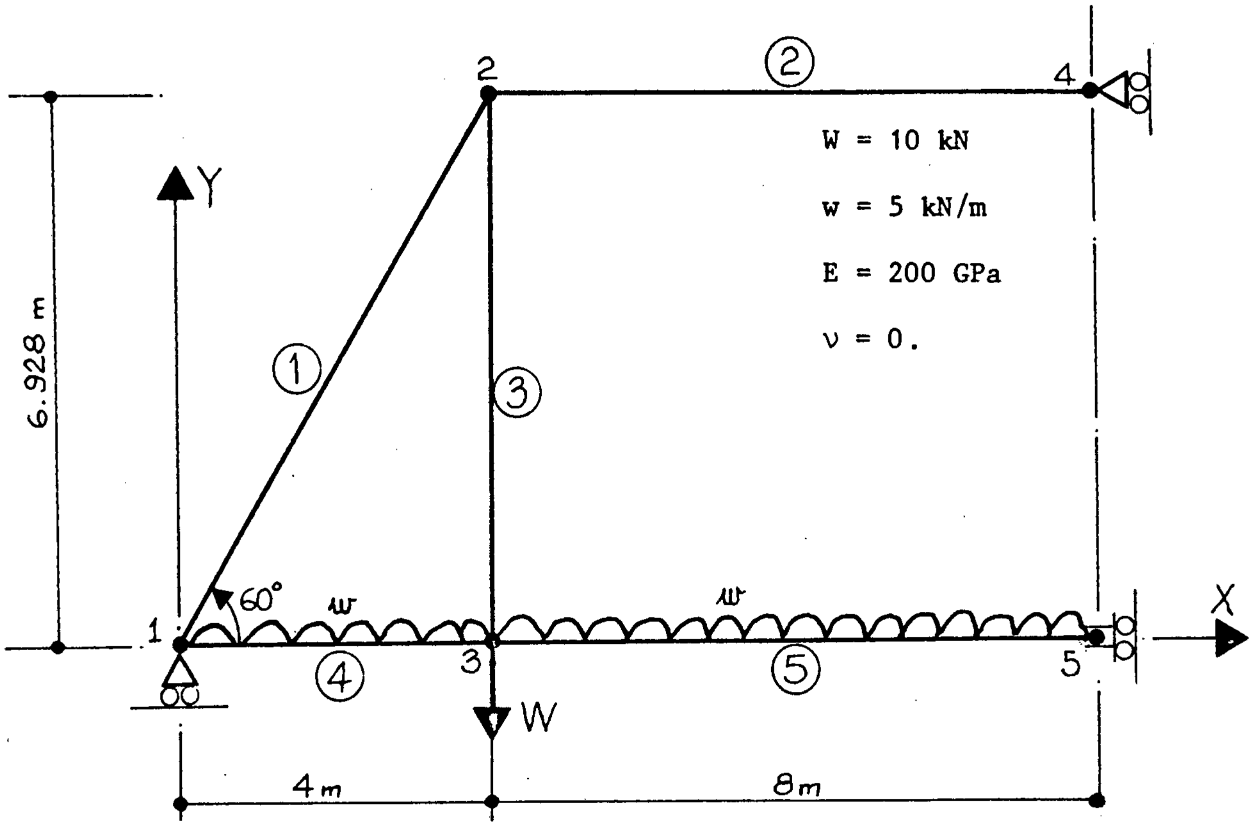
CE 5B17 : FINITE ELEMENT ANALYSIS

TIME : 3 hours

Note:

The student may refer to class notes and assignments.

1. The diagram shows half a plane symmetric frame structure, which is to be modelled using 5 elements.



Element 1,2 : linear bar ; $A = 5000 \text{ mm}^2$

3 : Bernoulli-Euler beam ; $A = 30000 \text{ mm}^2$

$I = 50 \times 10^6 \text{ mm}^4$

4,5 : linear Timoshenko beam ; $A = 125000 \text{ mm}^2$

$I = 2600 \times 10^6 \text{ mm}^4$

$\alpha = 1.2$

(i) Compute the <u>global</u> element stiffness matrices for elements 1, 3 and 4 .

(20)

(ii) Comment <u>briefly</u> on the relative stiffnesses contained in these element matrices.

(3)

(iii) Assemble elements 1 and 4 <u>only</u> into the global system stiffness matrix; show only what is necessary, i.e. nodes 1, 2 and 3 .

(5)

(iv) Compute the global system load vector.

(5)

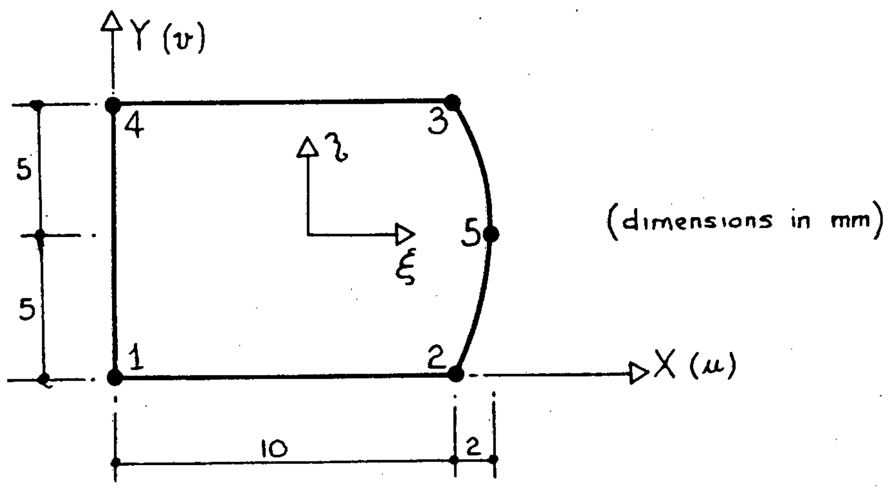
(v) Assume the following displacement solution (mm and rads) :

Node	u	v	θ
1	0.010	0.	-1.0×10^{-6}
3	0.012	-0.001	-0.5×10^{-6}

Using this solution compute the shear force and bending diagrams for element 4 .

(12)

2. A 5-noded quadrilateral plane stress element is shown below. Side 2-5-3 is curved.



(i) Show that the Jacobian matrix for this element is :

$$J_{11} = 6 - \eta^2 \qquad J_{12} = 0$$

$$J_{21} = -2\eta(1 + \xi) \qquad J_{22} = 5$$

(10)

(ii) For a thickness $t = 1$ mm , compute the volume of the element using exact numerical integration.

(5)

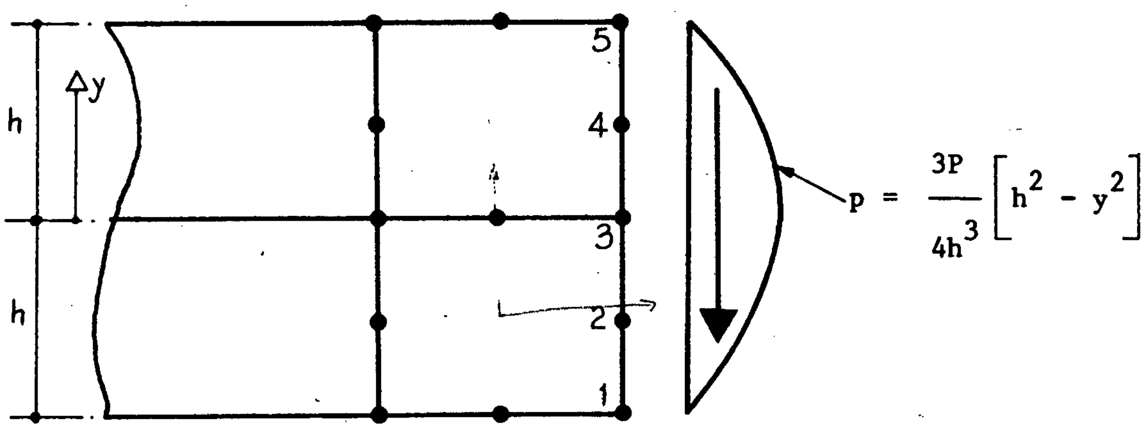
(iii) Show that the normal strain along the side 3-4 is a constant with magnitude

$$\varepsilon_{xx} = 0.1\ (u_3 - u_4)$$

(10)

Note: You may request the element shape functions if you do not have them in your notes.

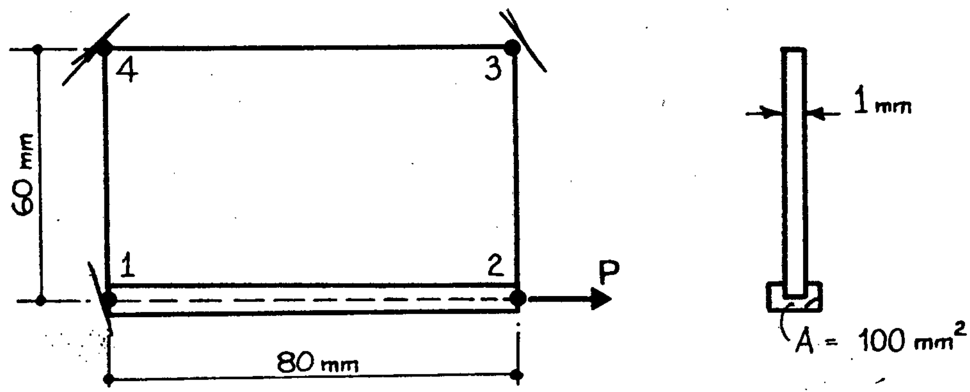
3. A parabolic shear load is applied across the edges of two 8-node quadrilateral elements as shown in the diagram.



Show that the equivalent nodal load at node 3 is (11/40)P .

(15)

4. A 4-node quadrilateral element and a 2-node bar element together support a point load P = 6000N . Nodes 1, 3 and 4 are fully restrained, and node 2 can only move horizontally.



The following additional information is given :

$$\frac{\partial N_2}{\partial x} = \frac{1}{16}(1-\eta) \quad ; \quad \frac{\partial N_2}{\partial y} = -\frac{1}{12}(1+\xi)$$

$$\underset{\sim}{J} = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix} \quad ; \quad E = 300\ \text{GPa} \ ; \quad \nu = 0.3$$

Compute the horizontal displacement at node 2.

(15)

UNIVERSITY OF CAPE TOWN

DEPARTMENT OF CIVIL ENGINEERING

COURSE CE 5D5 : ADVANCED HYDRAULIC STRUCTURES

There is no written examination for this course.

The result will be assessed from two equal value projects.

Project 1 : Analysis of side channel spillway.

Project 2 : Analysis of side weir overflow.

September 1985.

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(4 PAGES)

UNIVERSITY OF CAPE TOWN

DEPARTMENT OF CIVIL ENGINEERING

M.Sc. in CIVIL ENGINEERING

UNIVERSITY EXAMINATION : JULY 1986

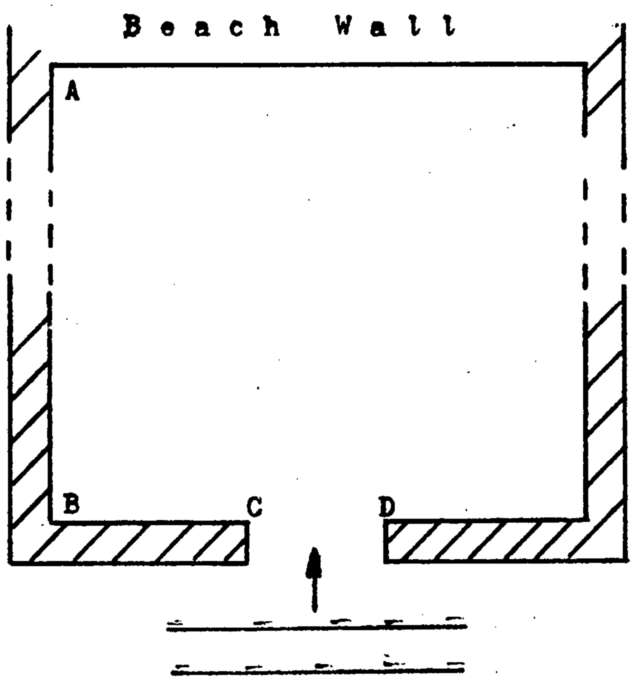
CIV 516F : COASTAL HYDRAULICS

ALL questions may be attempted. Time: 4 hours

Constants

Sea water density = 1025 kg/m^3
Sea water height = 10 kN/m^3

QUESTION 1



The diagram shows a rectangular shaped harbour with an entrance width of 174 m (CD) and side arm lengths of 261 m (BC). The harbour dimension to the beach wall is about 1,65 km (AB). If wave crests are approaching parallel to BC with a period of 10 seconds and a deep water wave height, H_o , of 4 metres, estimate the wave heights at 5 selected points along the wall AB and also at the harbour centre point. Give the angularity of the wave at the mid point of AB. Ignore refraction and reflection within the harbour, and take the water depth in the vicinity of the breakwater ends as 8,7 m. A selection of diffraction charts are available, ask for the one you need.

QUESTION 2

The standard alignment chart is attached, and a new blank line has been inserted at the bottom of the page. This line is to be used for determining values of u_{max} , the maximum horizontal orbital velocity at the bed, according to the Airy theory. If

$$u^*_{max} = \frac{u_{max}}{\pi H/T}$$

is to be the dimensionless form of the variable on this line, mark off the positions of the u^*_{max} values given in the following list :

u^*_{max} =	0,01	1,00	5,00
	0,10	2,00	6,00
	0,20	3,00	8,00
	0,50	4,00	10,00

Note that H is the local wave height throughout. Suggest a small change in the line label which would permit the scale to be used for maximum horizontal acceleration values. Use the line to solve the following problem.

QUESTION 2 (continued)

A swell of 10 second period with a deep water wave height $H_o = 3{,}2$ m approaches a beach with the wave crests parallel to the shore. Plot the value of u_{max} at the bed versus depth from deep water beachwards. Suggested values for depth are:

73m 56m 31m 16m 5m

If the beach sediment is mobile when the bed velocity exceeds 1 m/s, find the depth when this first occurs and check whether this is outside the breaker zone.

(1 diagram attached)

QUESTION 3

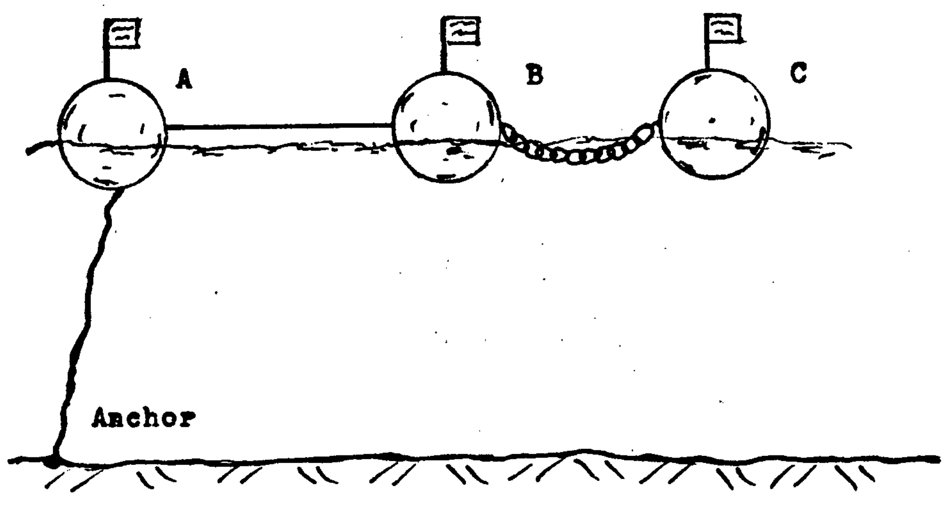
(a) The attached diagram shows a wave height record, metres versus seconds. Use the diagram to find the following :-

i. significant wave height by definition
ii. significant wave height by CERC method
iii. significant wave height by the Draper method.

(1 diagram attached)

(b) A wave rider buoy is a type of accelerometer device used for wave height measurements. If such an instrument requires calibration in a laboratory, for a range of typical sea wave periods and wave heights up to 10 m, suggest a way of doing this.

(c)



The diagram shows a device anchored to the sea bed outside the surf zone, where A, B and C are identical spherical buoys with visible flags on the top. A is joined to B with a nylon (lightweight) cord

QUESTION 3(c) (continued)

B is joined to C with a fairly heavy catenary chain. Suggest how an observer on the beach could use this device to take a coastal measurement, outline the detailed procedure and specify any beach instrument needed. State whether the device would need calibration.

QUESTION 4

A sea platform consists of a square concrete slab positioned horizontally on four cylindrical vertical piles, each placed at a corner, the slab side being parallel to the local wave crest. The pile diameter is 1 m, the total pile height above the sea bed is 6,4 m, and the slab dimensions are sides of 5 m with a thickness of 200 mm. The local wave characteristics are height 2 m, length 100 m, and period 12 s, the local water depth being 8 m.

(a) Considering the central 1 m high slice of any pile, calculate the horizontal forces per metre due to velocity and acceleration and by plotting these throughout one wave period identify the maximum force and the timing of its occurrence. Check that the velocity and acceleration distributions over the height of the pile are reasonably constant and thus estimate the total force on one pile.

Take $C_D = 1,2$ and $C_M = 2,0$.

(b) Estimate the maximum vertical force on the slab due to wave action.

Take $C_D = 1,0$ and $C_M = 1,8$.

QUESTION 5

In a study of wave penetration into a bay, the 9 m, 8 m and 7 m sea bed contours are approximated by three straight lines with contained angles of 12 degrees as shown on the attached page. An incoming wave orthogonal, 10 second period, impinges on the 9 m contour at an angle of 50 degrees as shown. With the usual approximations obtain by trial the angle at which the emerging orthogonal cuts the 7 m contour. Take the step lines on the 8,5 m and 7,5 m lines.

(1 diagram attached).

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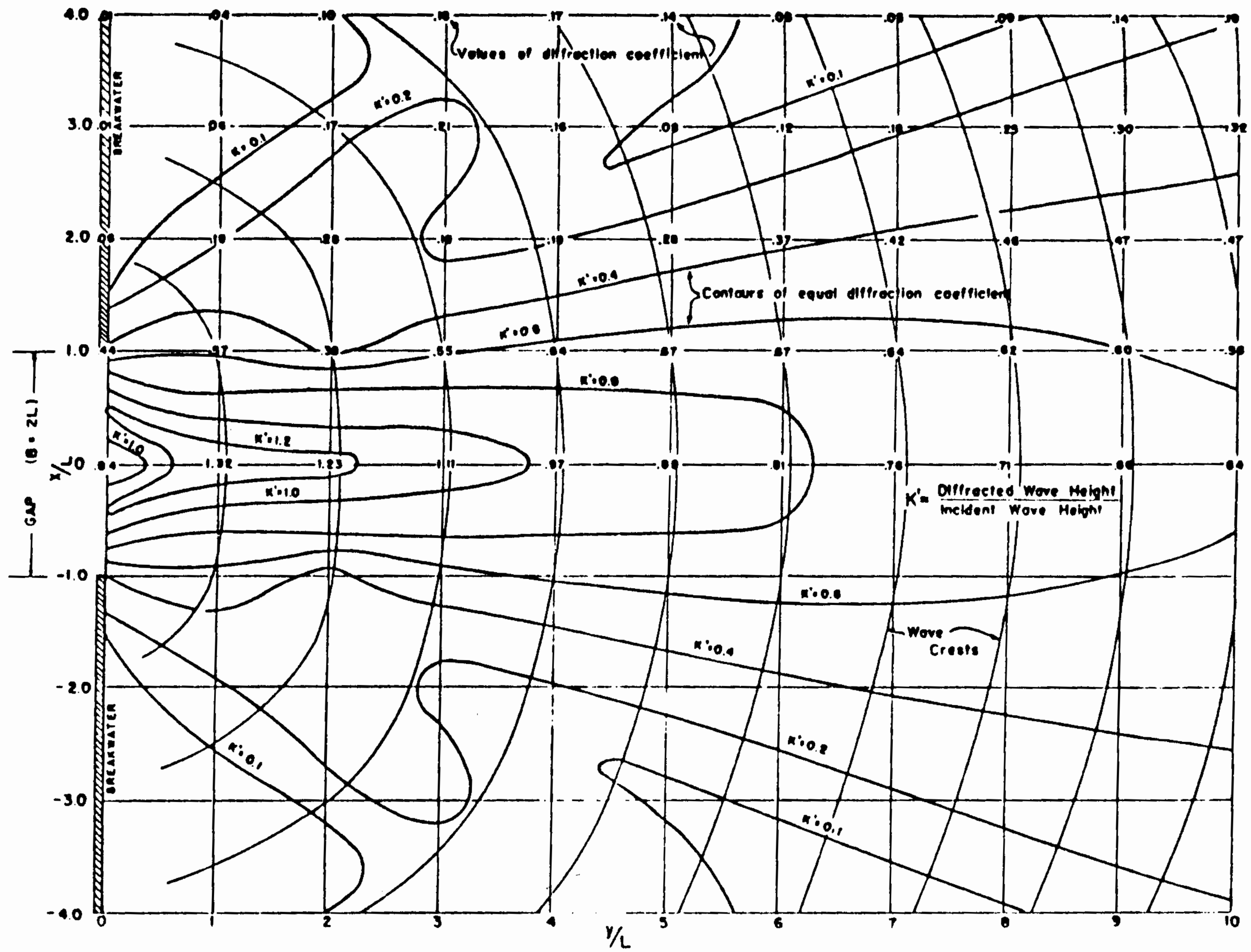
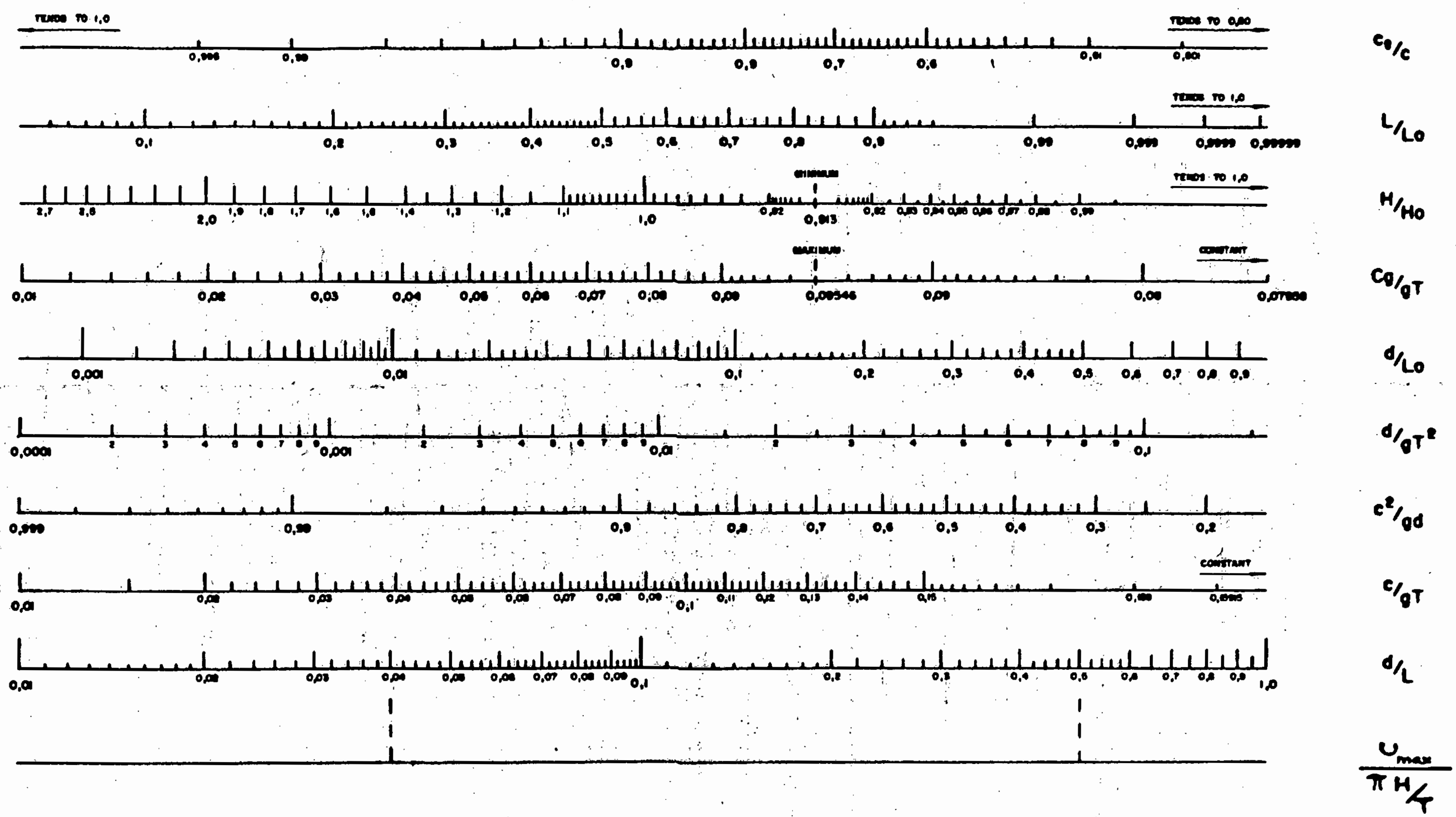


Figure 2-42. Generalized diffraction diagram for a breakwater gap width of two wavelengths (B/L = 2).



ALIGNMENT CHART FOR DIMENSIONLESS BASIC WAVE RELATIONSHIPS

PREPARED BY W J T
1977

Coastal Engineering Course

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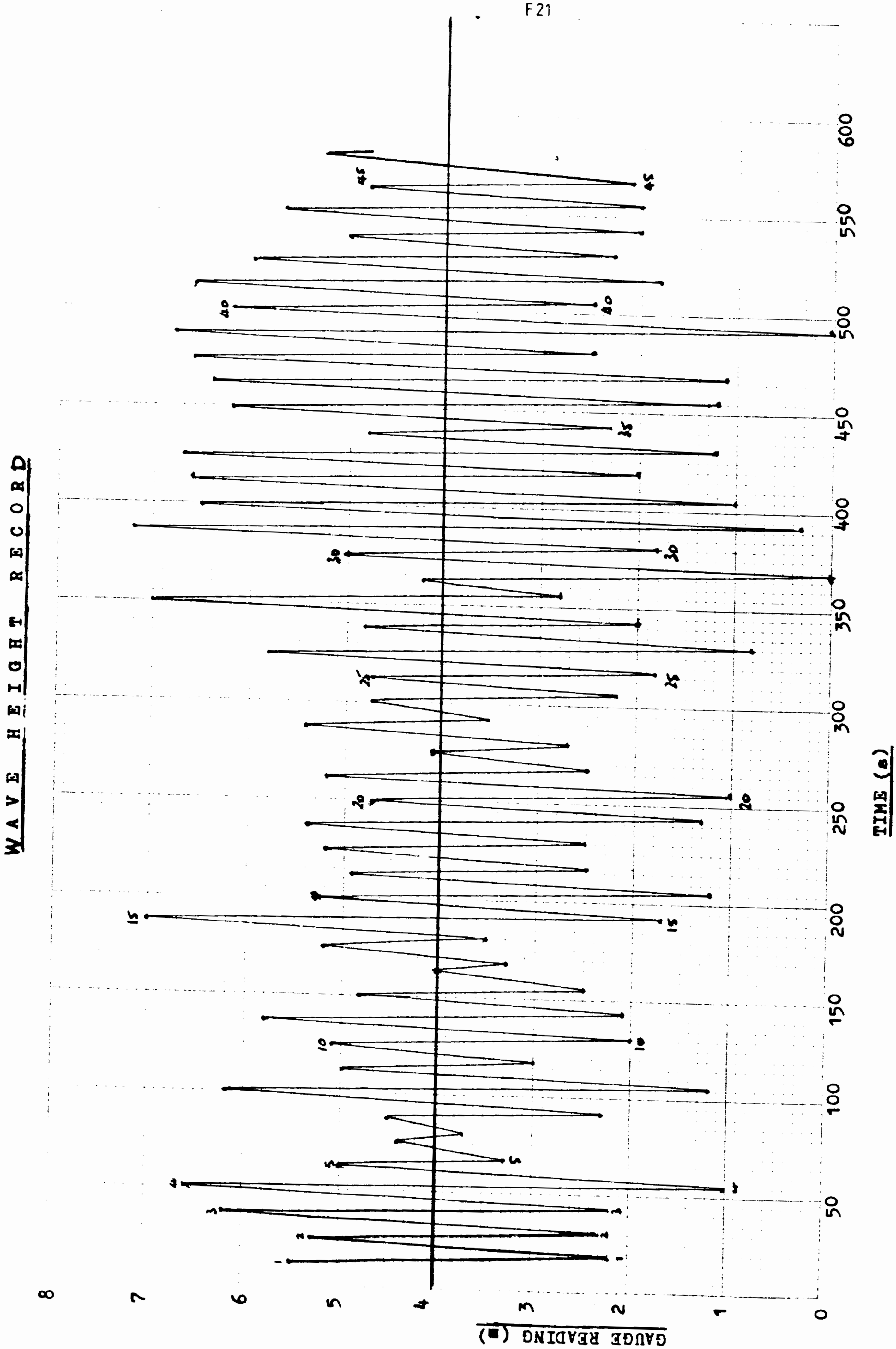

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